

# Mathematics

Advanced GCE A2 7890 - 2

Advanced Subsidiary GCE AS 3890 - 2

## Mark Schemes for the Units

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## CONTENTS

**Advanced GCE Mathematics (7890)**  
**Advanced GCE Pure Mathematics (7891)**  
**Advanced GCE Further Mathematics (7892)**

**Advanced Subsidiary GCE Mathematics (3890)**  
**Advanced Subsidiary GCE Pure Mathematics (3891)**  
**Advanced Subsidiary GCE Further Mathematics (3892)**

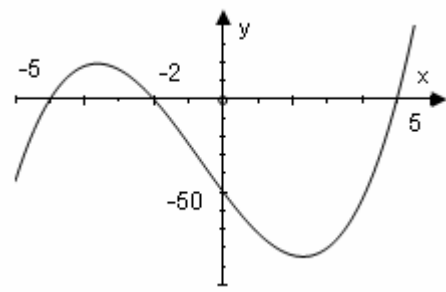
### MARK SCHEMES FOR THE UNITS

#### **Unit/ContentPage**

4721 Core Mathematics 1	1
4722 Core Mathematics 2	6
4723 Core Mathematics 3	10
4724 Core Mathematics 4	13
4725 Further Pure Mathematics 1	17
4726 Further Pure Mathematics 2	21
4727 Further Pure Mathematics 3	24
4728 Mechanics 1	31
4729 Mechanics 2	33
4730 Mechanics 3	35
4731 Mechanics 4	38
4732 Probability & Statistics 1	42
4733 Probability & Statistics 2	46
4734 Probability & Statistics 3	487
4735 Statistics 4	510
4736 Decision Mathematics 1	543
4737 Decision Mathematics 2	598
Grade Thresholds	643

# 4721 Core Mathematics 1

1 (i)	$n = -2$	B1 1
(ii)	$n = 3$	B1 1
(iii)	$n = \frac{3}{2}$	M1 $\sqrt{4^3}$ or $64^{\frac{1}{2}}$ or $\left(4^{\frac{1}{2}}\right)^3$ or $(4^3)^{\frac{1}{2}}$ or $4 \times \sqrt{4}$ with brackets correct if used A1 2
2 (i)	$y = (x-2)^2$	M1 $y = (x \pm 2)^2$ A1 2
(ii)	$y = -(x^3 - 4)$	B1 oe 1
3 (i)	$\sqrt{2 \times 100} = 10\sqrt{2}$	B1 1
(ii)	$\frac{12}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$	B1 1
(iii)	$10\sqrt{2} - 3\sqrt{2} = 7\sqrt{2}$	M1 Attempt to express $5\sqrt{8}$ in terms of $\sqrt{2}$ A1 2
4	$y = x^{\frac{1}{2}}$ $2y^2 - 7y + 3 = 0$ $(2y-1)(y-3) = 0$ $y = \frac{1}{2}, y = 3$ $x = \frac{1}{4}, x = 9$	M1* Use a substitution to obtain a quadratic or factorise into 2 brackets each containing $x^{\frac{1}{2}}$ M1dep Correct method to solve a quadratic A1 M1 Attempt to square to obtain $x$ A1 SR If first M1 not gained and 3 and $\frac{1}{2}$ given as final answers, award B1 5

<p>5</p> $\frac{dy}{dx} = 4x^{-\frac{1}{2}} + 1$ $= 4\left(\frac{1}{\sqrt{9}}\right) + 1$ $\frac{dy}{dx} = \frac{7}{3}$	<p>M1 Attempt to differentiate</p> <p>A1 <math>kx^{-\frac{1}{2}}</math></p> <p>A1</p> <p>M1 Correct substitution of <math>x = 9</math> into their</p> <p>A1 <math>\frac{7}{3}</math> only</p> <p><b>5</b></p>
<p>6 (i) <math>(x-5)(x+2)(x+5)</math></p> $= (x^2 - 3x - 10)(x+5)$ $= x^3 + 2x^2 - 25x - 50$	<p>B1 <math>x^2 - 3x - 10</math> or <math>x^2 + 7x + 10</math> or <math>x^2 - 25</math> seen</p> <p>M1 Attempt to multiply a quadratic by a linear factor</p> <p>A1</p> <p><b>3</b></p>
<p>(ii)</p> 	<p>B1 +ve cubic with 3 roots (not 3 line segments)</p> <p>B1✓ (0, -50) labelled or indicated on y-axis</p> <p>B1 (-5, 0), (-2, 0), (5, 0) labelled or indicated on x-axis and no other x- intercepts</p> <p><b>3</b></p>
<p>7 (i) <math>8 &lt; 3x - 2 &lt; 11</math></p> $10 < 3x < 13$ $\frac{10}{3} < x < \frac{13}{3}$	<p>M1 2 equations or inequalities both dealing with all 3 terms resulting in <math>a &lt; kx &lt; b</math></p> <p>A1 10 and 13 seen</p> <p>A1</p> <p><b>3</b></p>
<p>(ii) <math>x(x+2) \geq 0</math></p> $x \geq 0, x \leq -2$	<p>M1 Correct method to solve a quadratic</p> <p>A1 0, -2</p> <p>M1 Correct method to solve inequality</p> <p>A1</p> <p><b>4</b></p>

8 (i)	$\frac{dy}{dx} = 3x^2 - 2kx + 1$	B1 One term correct B1 Fully correct
		<span style="border: 1px solid black; padding: 2px;">2</span>
(ii)	$3x^2 - 2kx + 1 = 0$ when $x = 1$  $3 - 2k + 1 = 0$  $k = 2$	M1 their $\frac{dy}{dx} = 0$ soi M1 $x = 1$ substituted into their $\frac{dy}{dx} = 0$ A1✓
		<span style="border: 1px solid black; padding: 2px;">3</span>
(iii)	$\frac{d^2y}{dx^2} = 6x - 4$  When $x = 1$ , $\frac{d^2y}{dx^2} > 0 \therefore$ min pt	M1 Substitutes $x = 1$ into their $\frac{d^2y}{dx^2}$ and looks at sign A1 States minimum CWO
		<span style="border: 1px solid black; padding: 2px;">2</span>
(iv)	$3x^2 - 4x + 1 = 0$  $(3x - 1)(x - 1) = 0$  $x = \frac{1}{3}, x = 1$  $x = \frac{1}{3}$	M1 their $\frac{dy}{dx} = 0$ M1 correct method to solve 3-term quadratic  A1 WWW at any stage
		<span style="border: 1px solid black; padding: 2px;">3</span>

<p><b>9 (i)</b></p> $(x-2)^2 + (y-1)^2 = 100$ $x^2 + y^2 - 4x - 2y - 95 = 0$	<p><b>B1</b> <math>(x-2)^2</math> and <math>(y-1)^2</math> seen</p> <p><b>B1</b> <math>(x \pm 2)^2 + (y \pm 1)^2 = 100</math></p> <p><b>B1</b> correct form</p> <p style="text-align: center;"><b>3</b></p>
<p><b>(ii)</b></p> $(5-2)^2 + (k-1)^2 = 100$ $(k-1)^2 = 91 \quad \text{or} \quad k^2 - 2k - 90 = 0$ $k = 1 + \sqrt{91}$	<p><b>M1</b> <math>x = 5</math> substituted into their equation</p> <p><b>A1</b> correct, simplified quadratic in <math>k</math> (or <math>y</math>) obtained</p> <p><b>A1</b> cao</p> <p style="text-align: center;"><b>3</b></p>
<p><b>(iii)</b> distance from <math>(-3, 9)</math> to <math>(2, 1)</math></p> $= \sqrt{(2 - (-3))^2 + (1 - 9)^2}$ $= \sqrt{25 + 64}$ $= \sqrt{89}$ $\sqrt{89} < 10 \quad \text{so point is inside}$	<p><b>M1</b> Uses <math>(x_2 - x_1)^2 + (y_2 - y_1)^2</math></p> <p><b>A1</b></p> <p><b>B1</b> compares their distance with 10 and makes consistent conclusion</p> <p style="text-align: center;"><b>3</b></p>
<p><b>(iv)</b> gradient of radius <math>= \frac{9-1}{8-2}</math></p> $= \frac{4}{3}$ <p>gradient of tangent <math>= -\frac{3}{4}</math></p> $y-9 = -\frac{3}{4}(x-8)$ $y-9 = -\frac{3}{4}x + 6$ $y = -\frac{3}{4}x + 15$	<p><b>M1</b> uses <math>\frac{y_2 - y_1}{x_2 - x_1}</math></p> <p><b>A1</b> oe</p> <p><b>B1✓</b> oe</p> <p><b>M1</b> correct equation of straight line through <math>(8, 9)</math>, any non-zero gradient</p> <p><b>A1</b> oe 3 term equation</p> <p style="text-align: center;"><b>5</b></p>

<b>10 (i)</b> $2(x^2 - 3x) + 11$ $= 2\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 11$ $= 2\left(x - \frac{3}{2}\right)^2 + \frac{13}{2}$	<b>B1</b> $p = 2$ <b>B1</b> $q = -\frac{3}{2}$ <b>M1</b> $r = 11 - 2q^2$ or $\frac{11}{2} - q^2$ <b>A1</b> $r = \frac{13}{2}$ <b>4</b>
<b>(ii)</b> $\left(\frac{3}{2}, \frac{13}{2}\right)$	<b>B1√</b> <b>B1√</b> <b>2</b>
<b>(iii)</b> $36 - 4 \times 2 \times 11$ $= -52$	<b>M1</b> uses $b^2 - 4ac$ <b>A1</b> <b>2</b>
<b>(iv)</b> 0 real roots	<b>B1</b> cao <b>1</b>
<b>(v)</b> $2x^2 - 6x + 11 = 14 - 7x$ $2x^2 + x - 3 = 0$ $(2x + 3)(x - 1) = 0$ $x = -\frac{3}{2}, x = 1$ $y = \frac{49}{2}, y = 7$	<b>M1*</b> substitute for $x/y$ or attempt to get an equation in 1 variable only <b>A1</b> obtain correct 3 term quadratic <b>M1dep</b> correct method to solve 3 term quadratic <b>A1</b> <b>A1</b> <b>SR</b> If A0 A0, one correct pair of values, spotted or from correct factorisation <b>www B1</b> <b>5</b>



# 4722 Core Mathematics 2

<p>1      <math>(2 - 3x)^6 = 2^6 + 6 \cdot 2^5 \cdot (-3x) + 15 \cdot 2^4 \cdot (-3x)^2</math>  <math>\qquad \qquad \qquad = 64 - 576x + 2160x^2</math></p>	<p><b>M1</b>    Attempt (at least) first two terms - product of binomial coefficient and powers of 2 and (-  <math>3x</math>)  <b>A1</b>    Obtain <math>64 - 576x</math>  <b>M1</b>    Attempt third term - binomial coefficient and powers of 2 and (-<math>3x</math>)  <b>A1</b>    Obtain <math>2160x^2</math></p>
<p><b>OR</b></p>	<p><b>M1</b>    Attempt expansion involving all 6 brackets  <b>A1</b>    Obtain 64  <b>A1</b>    Obtain <math>-576x</math>  <b>A1</b>    Obtain <math>2160x^2</math></p>
<p><b>SR</b> if the expansion is attempted in descending order, and the required terms are never seen, then <b>B1 B1 B1</b> for <math>4860x^4, -2916x^5, 729x^6</math></p>	

4

<p>2 (i) <math>u_2 = \frac{2}{3}</math>  <math>u_3 = \frac{-1}{2}</math>  <math>u_4 = 3</math></p>	<p><b>B1</b>    Obtain correct <math>u_2</math>  <b>B1</b>✓ Obtain correct <math>u_3</math> from their <math>u_2</math>  <b>B1</b>✓ Obtain correct <math>u_4</math> from their <math>u_3</math></p>
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3

<p>(ii) sequence is periodic / cyclic / repeating</p>	<p><b>B1</b>    Any equivalent comment</p>
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1

<p>3 (i) <math>\frac{1}{2} \times 8^2 \times \theta = 48</math>  Hence <math>\theta = 1.5</math> radians</p>	<p><b>M1</b>    State or imply <math>(\frac{1}{2}) 8^2 \theta = 48</math>  <b>A1</b>    Obtain <math>\theta = 1.5</math> (or <math>0.477\pi</math>), or equiv</p>
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2

<p>(ii) area = <math>48 - \frac{1}{2} \times 8^2 \times \sin 1.5</math>  <math>= 48 - 31.9</math>  <math>= 16.1</math></p>	<p><b>M1</b>*    Attempt area of <math>\Delta</math> using <math>(\frac{1}{2}) 8^2 \sin \theta</math>  <b>M1d</b>*    Attempt <math>48 -</math> area of <math>\Delta</math>  <b>A1</b>    Obtain <math>16.1 \text{ cm}^2</math></p>
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3

<p>4 (i) <math>f(3) = 27a - 36 - 21a + 12 = 0</math>  <math>6a = 24</math>  <math>a = 4</math></p>	<p><b>M1</b>*    Attempt <math>f(3)</math>  <b>M1d</b>*    Equate attempt at <math>f(3)</math> to 0 and attempt to solve  <b>A1</b>    Obtain <math>a = 4</math></p>
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**OR**


**M1**\*    Attempt complete division / matching coeffs  
**M1d**\*    Equate remainder to 0  
**A1**    Obtain  $a = 4$

3

<p>(ii) <math>f(-2) = -32 - 16 + 56 + 12</math>  <math>= 20</math></p>	<p><b>M1</b>    Attempt <math>f(-2)</math>  <b>A1</b>✓ Obtain 20 (or <math>6a - 4</math>, following their <math>a</math>)</p>
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2

<p>5 (i) <math>\int xdy = \int ((y-3)^2 - 2)dy</math>  <math>= \int (y^2 - 6y + 7)dy</math> A.G.  <math>3 + \sqrt{(2+2)} = 5, 3 + \sqrt{(14+2)} = 7</math></p>	<p><b>B1</b> Show <math>x = y^2 - 6y + 7</math> convincingly  <b>B1</b> State or imply that required area <math>= \int xdy</math>  <b>B1</b> Use <math>x = 2, 14</math> to show new limits of <math>y = 5, 7</math>  <b>3</b></p>
<p>(ii) <math>\left[\frac{1}{3}y^3 - 3y^2 + 7y\right]_5^7</math>  term  <math>= ({}^{343}/_3 - 147 + 49) - ({}^{125}/_3 - 75 + 35)</math>  <math>= 16\frac{1}{3} - 1\frac{2}{3}</math>  <math>= 14\frac{2}{3}</math></p>	<p><b>M1</b> Integration attempt, with at least one correct  <b>A1</b> All three terms correct  <b>M1</b> Attempt <math>F(7) - F(5)</math>  <b>A1</b> Obtain <math>14\frac{2}{3}</math>, or exact equiv  <b>4</b></p>
<p>6 (i) <math>ABC = 360 - (150 + 110) = 100^\circ</math> A.G.</p>	<p><b>B1</b> Show convincingly that angle <math>ABC</math> is <math>100^\circ</math>  <b>1</b></p>
<p>(ii) <math>CA^2 = 15^2 + 27^2 - 2 \times 15 \times 27 \times \cos 100^\circ</math>  <math>= 1094.655\dots</math>  <math>CA = 33.1</math></p>	<p><b>M1</b> Attempt use of correct cosine rule  <b>A1</b> Obtain 33.1 km  <b>2</b></p>
<p>(iii) <math>\frac{\sin C}{15} = \frac{\sin 100}{33.1}</math> or <math>\frac{\sin A}{27} = \frac{\sin 100}{33.1}</math>    <math>C = 26.5^\circ</math> <math>A = 53.5^\circ</math>  Hence bearing is <math>263^\circ</math></p>	<p><b>M1</b> Attempt use of sine rule to find angle <math>C</math> or <math>A</math> (or equiv using cosine rule)  <b>A1</b> Correct unsimplified eqn, following their <math>CA</math>  <b>A1</b> Obtain <math>C = 26.5^\circ</math> or <math>A = 53.5^\circ</math> (allow <math>53.4^\circ</math>)  <b>A1</b> Obtain 263 or 264 (or <math>290^\circ -</math> their angle <math>C / 210 +</math> their angle <math>A</math>)  <b>4</b></p>
<p>7 (a) <math>\int (x^5 - x^4 + 5x^3)dx</math>    <math>= \frac{1}{6}x^6 - \frac{1}{5}x^5 + \frac{5}{4}x^4 (+c)</math></p>	<p><b>M1</b> Expand brackets and attempt integration, or other valid integration attempt  <b>A1</b> Obtain at least one correct term  <b>A1</b> Obtain a fully correct expression  <b>B1</b> For <math>+c</math>, and no <math>\int</math> or <math>dx</math> (can be given in (b)(i) if not given here)  <b>4</b></p>
<p>(b) (i) <math>-6x^{-3} (+c)</math></p>	<p><b>M1</b> Obtain integral of the form <math>kx^{-3}</math>  <b>A1</b> Obtain <math>-6x^{-3} (+c)</math>  <b>2</b></p>
<p>(ii) <math>\left[-6x^{-3}\right]_2^\infty</math>  <math>= \frac{3}{4}</math></p>	<p><b>B1*</b> State or imply that <math>F(\infty) = 0</math> (for <math>kx^n, n &lt; -1</math>)  <b>B1d*</b> Obtain <math>\frac{3}{4}</math> (or equiv)  <b>2</b></p>

8 (i)		<p><b>M1</b> Attempt sketch of exponential graph (1<sup>st</sup> quad) - if seen in 2<sup>nd</sup> quad must be approx correct</p> <p><b>A1</b> Correct graph in both quadrants</p> <p><b>B1</b> State or imply (0, 2) only</p> <p style="text-align: center;"><b>3</b></p>
(ii)	$8^x = 2 \times 3^x$ $\log_2 8^x = \log_2 (2 \times 3^x)$ $x \log_2 8 = \log_2 2 + x \log_2 3$ $3x = 1 + x \log_2 3$ $x(3 - \log_2 3) = 1, \text{ hence } x = \frac{1}{3 - \log_2 3} \text{ A.G.}$	<p><b>M1</b> Form equation in <math>x</math> and take logs (to any consistent base, or no base) – could use <math>\log_8</math></p> <p><b>M1</b> Use <math>\log a^b = b \log a</math></p> <p><b>M1</b> Use <math>\log ab = \log a + \log b</math>, or equiv with <math>\log^{a/b}</math></p> <p><b>M1</b> Use <math>\log_2 8 = 3</math></p> <p><b>A1</b> Show given answer correctly</p>
<b>OR</b>	$8^x = 2 \times 3^x$ $2^{3x} = 2 \times 3^x$ $2^{(3x-1)} = 3^x$ $\log_2 2^{(3x-1)} = \log_2 3^x$ $(3x-1) \log_2 2 = x \log_2 3$ $x(3 - \log_2 3) = 1, \text{ hence } x = \frac{1}{3 - \log_2 3} \text{ A.G.}$	<p><b>M1</b> Use <math>8^x = 2^{3x}</math></p> <p><b>M1</b> Attempt to rearrange equation to <math>2^k = 3^x</math></p> <p><b>M1</b> Take logs (to any base)</p> <p><b>M1</b> Use <math>\log a^b = b \log a</math></p> <p><b>A1</b> Show given answer correctly</p> <p style="text-align: center;"><b>5</b></p>
9 (a) (i)	$2 \sin x \frac{\sin x}{\cos x} - 5 = \cos x$ $2 \sin^2 x - 5 \cos x = \cos^2 x$ $2 - 2 \cos^2 x - 5 \cos x = \cos^2 x$ $3 \cos^2 x + 5 \cos x - 2 = 0$	<p><b>M1</b> Use <math>\tan x \equiv \frac{\sin x}{\cos x}</math></p> <p><b>M1</b> Use <math>\sin^2 x \equiv 1 - \cos^2 x</math></p> <p><b>A1</b> Show given equation convincingly</p> <p style="text-align: center;"><b>3</b></p>
(ii)	$(3 \cos x - 1)(\cos x + 2) = 0$ $\cos x = \frac{1}{3}$ $x = 1.23 \text{ rad}$ $x = 5.05 \text{ rad}$	<p><b>M1</b> Attempt to solve quadratic in <math>\cos x</math></p> <p><b>M1</b> Attempt to find <math>x</math> from root(s) of quadratic</p> <p><b>A1</b> Obtain 1.23 rad or <math>70.5^\circ</math></p> <p><b>A1</b> Obtain 5.05 rad or <math>289^\circ</math> (or <math>2\pi / 360^\circ</math> - their solution)</p> <p><b>SR: B1 B1</b> for answer(s) only</p> <p style="text-align: center;"><b>4</b></p>
(b)	$0.5 \times 0.25x \{ \cos 0 + 2(\cos 0.25 + \cos 0.5 + \cos 0.75) + \cos 1 \}$ $\approx 0.837$	<p><b>M1</b> Attempt <math>y</math>-coords for at least 4 of the correct 5 <math>x</math>-coords</p> <p><b>M1</b> Use correct trapezium rule, any <math>h</math>, for their <math>y</math> values to find area between <math>x = 0</math> and <math>x = 1</math></p> <p><b>M1</b> Correct <math>h</math> (soi) for their <math>y</math> values</p> <p><b>A1</b> Obtain 0.837</p> <p style="text-align: center;"><b>4</b></p>

<b>10 (i)</b> $u_{15} = 2 + 14 \times 0.5$ $= 9 \text{ km}$	<b>M1</b> Attempt use of $a + (n - 1)d$ <b>A1</b> Obtain 9 km <b>2</b>
<b>(ii)</b> $u_{20} = 2 \times 1.1^{19} = 12.2$ $u_{19} = 2 \times 1.1^{18} = 11.1$	<b>B1</b> State, or imply, $r = 1.1$ <b>M1</b> Attempt $u_{20}$ , using $ar^{n-1}$ <b>A1</b> Obtain $u_{20} = 12.2$ , and obtain $u_{19} = 11.1$
<b>OR</b>	<b>B1</b> State, or imply, $r = 1.1$ <b>M1</b> Attempt to solve $ar^{n-1} = 12$ <b>A1</b> Obtain $n = 20$ (allow $n \geq 20$ ) <b>3</b>
<b>(iii)</b> $\frac{2(1.1^n - 1)}{(1.1 - 1)} > 200$ $1.1^n > 11$ $n > \frac{\log 11}{\log 1.1}$ $n > 25.2$ ie Day 26	<b>B1</b> State or imply $S_N = \frac{2(1.1^n - 1)}{(1.1 - 1)}$ <b>M1</b> Link (any sign) their attempt at $S_N$ (of a GP) to 200 and attempt to solve <b>A1</b> Obtain 26, or 25.2 or better <b>A1</b> Conclude $n = 26$ only, or equiv eg Day 26 <b>4</b>
<b>(iv)</b> swum = $2 \times 30 = 60 \text{ km}$ run = $\frac{1}{2} \times 30 \times (4 + 29 \times 0.5)$ $= 277.5 \text{ km}$ cycle = $\frac{2(1.1^{30} - 1)}{(1.1 - 1)}$ $= 329.0 \text{ km}$ total = 666 km	<b>B1</b> Obtain 60 km, or $2 \times 30 \text{ km}$ <b>M1</b> Attempt sum of AP, $d = 0.5$ , $a = 2$ , $n = 30$ <b>M1</b> Attempt sum of GP, $r = 1.1$ , $a = 2$ , $n = 30$ <b>A1</b> Obtain 666 or 667 km <b>4</b>

# 4723 Core Mathematics 3

<p>1 <u>Either</u>: Obtain <math>x = 0</math> Form linear equation with signs of <math>4x</math> and <math>3x</math> different State <math>4x - 5 = -3x + 5</math> Obtain <math>\frac{10}{7}</math> and no other non-zero value(s)</p>	<p><b>B1</b> ignoring errors in working <b>M1</b> ignoring other sign errors <b>A1</b> or equiv without brackets <b>A1</b> or exact equiv</p>												
<p><u>Or</u>: Obtain <math>16x^2 - 40x + 25 = 9x^2 - 30x + 25</math> Attempt solution of quadratic equation  Obtain <math>\frac{10}{7}</math> and no other non-zero value(s) Obtain 0</p>	<p><b>4</b> <b>B1</b> or equiv <b>M1</b> at least as far as factorisation or use of formula <b>A1</b> or exact equiv <b>B1</b> ignoring errors in working <b>4</b></p>												
<p>2 (i) Show graph indicating attempt at reflection in <math>y = x</math>  Show correct graph with <math>x</math>-coord 2 and <math>y</math>-coord <math>-3</math> indicated</p>	<p><b>M1</b> with correct curvature and crossing negative <math>y</math>-axis and positive <math>x</math>-axis  <b>A1</b> <b>2</b></p>												
<p>(ii) Show graph indicating attempt at reflection in <math>x</math>-axis  Show correct graph with <math>x</math>-coord <math>-3</math> indicated ... and <math>y</math>-coord <math>-4</math> indicated [SC: Incorrect curve earning M0 but both correct intercepts indicated</p>	<p><b>M1</b> with correct curvature and crossing each negative axis  <b>A1</b> <b>A1</b> B1] <b>3</b></p>												
<p>3 Attempt use of product rule Obtain <math>2x \ln x + x^2 \cdot \frac{1}{x}</math> Substitute <math>e</math> to obtain <math>3e</math> for gradient Attempt eqn of straight line with numerical gradient Obtain <math>y - e^2 = 3e(x - e)</math>  Obtain <math>y = 3ex - 2e^2</math></p>	<p><b>M1</b> ... + ... form <b>A1</b> or equiv <b>A1</b> or exact (unsimplified) equiv <b>M1</b> allowing approx values <b>A1</b> or equiv; following their gradient provided obtained by diffn attempt; allow approx values <b>A1</b> in terms of <math>e</math> now and in requested form <b>6</b></p>												
<p>4 (i) Differentiate to obtain form <math>kx(2x^2 + 9)^n</math> Obtain correct <math>10x(2x^2 + 9)^{\frac{3}{2}}</math> Equate to 100 and confirm <math>x = 10(2x^2 + 9)^{-\frac{3}{2}}</math></p>	<p><b>M1</b> any constant <math>k</math>; any <math>n &lt; \frac{5}{2}</math> <b>A1</b> or (unsimplified) equiv <b>A1</b> <b>AG</b>; necessary detail required <b>3</b></p>												
<p>(ii) Attempt relevant calculations with 0.3 and 0.4 Obtain at least one correct value  Obtain two correct values and conclude appropriately</p>	<p><b>M1</b> <b>A1</b></p> <table border="1" data-bbox="973 1702 1356 1814"> <thead> <tr> <th><math>x</math></th> <th><math>f(x)</math></th> <th><math>x - f(x)</math></th> <th><math>f'(x)</math></th> </tr> </thead> <tbody> <tr> <td>0.3</td> <td>0.3595</td> <td>-0.0595</td> <td>83.4</td> </tr> <tr> <td>0.4</td> <td>0.3515</td> <td>0.0485</td> <td>113.8</td> </tr> </tbody> </table> <p><b>A1</b> noting sign change or showing <math>0.3 &lt; f(0.3)</math> and <math>0.4 &gt; f(0.4)</math> or showing gradients either side of 100 <b>3</b></p>	$x$	$f(x)$	$x - f(x)$	$f'(x)$	0.3	0.3595	-0.0595	83.4	0.4	0.3515	0.0485	113.8
$x$	$f(x)$	$x - f(x)$	$f'(x)$										
0.3	0.3595	-0.0595	83.4										
0.4	0.3515	0.0485	113.8										

<b>(iii)</b> Obtain correct first iterate Carry out correct process Obtain 0.3553	<b>B1</b> <b>M1</b> finding at least 3 iterates in all <b>A1</b> answer required to exactly 4 dp
$[0.3 \rightarrow 0.35953 \rightarrow 0.35497 \rightarrow 0.35534 \rightarrow 0.35531;$ $0.35 \rightarrow 0.35575 \rightarrow 0.35528 \rightarrow 0.35532 (\rightarrow 0.35531);$ $0.4 \rightarrow 0.35146 \rightarrow 0.35563 \rightarrow 0.35529 \rightarrow 0.35532]$	
<b>5 (a)</b> Obtain expression of form $\frac{a \tan \alpha}{b + c \tan^2 \alpha}$	<b>M1</b> any non-zero constants $a, b, c$
State correct $\frac{2 \tan \alpha}{1 - \tan^2 \alpha}$	<b>A1</b> or equiv
Attempt to produce polynomial equation in $\tan \alpha$	<b>M1</b> using sound process
Obtain at least one correct value of $\tan \alpha$	<b>A1</b> $\tan \alpha = \pm \sqrt{\frac{4}{5}}$
Obtain 41.8	<b>A1</b> allow 42 or greater accuracy; allow 0.73
Obtain 138.2 and no other values between 0 and 180	<b>A1</b> allow 138 or greater accuracy
[SC: Answers only 41.8 or ... B1; 138.2 or ... and no others B1]	<b>6</b>
<b>(b)(i)</b> State $\frac{7}{6}$	<b>B1</b>
<b>1</b>	
<b>(ii)</b> Attempt use of identity linking $\cot^2 \beta$ and $\operatorname{cosec}^2 \beta$	<b>M1</b> or equiv retaining exactness; condone sign errors
Obtain $\frac{13}{36}$	<b>A1</b> or exact equiv
<b>2</b>	
<b>6</b> Integrate $k_1 e^{nx}$ to obtain $k_2 e^{nx}$	<b>M1</b> any constants involving $\pi$ or not; any $n$
Obtain correct indefinite integral of their $k_1 e^{nx}$	<b>A1</b>
Substitute limits to obtain $\frac{1}{6} \pi (e^3 - 1)$ or $\frac{1}{6} (e^3 - 1)$	<b>A1</b> or exact equiv perhaps involving $e^0$
Integrate $k(2x - 1)^n$ to obtain $k'(2x - 1)^{n+1}$	<b>M1</b> any constants involving $\pi$ or not; any $n$
Obtain correct indefinite integral of their $k(2x - 1)^n$	<b>A1</b>
Substitute limits to obtain $\frac{1}{18} \pi$ or $\frac{1}{18}$	<b>A1</b> or exact equiv
Apply formula $\int \pi y^2 dx$ at least once	<b>B1</b> for $y = e^{3x}$ and/or $y = (2x - 1)^4$
$y^2$ Subtract, correct way round, attempts at volumes	<b>M1</b> allow with $\pi$ missing but must involve
Obtain $\frac{1}{6} \pi e^3 - \frac{2}{9} \pi$	<b>A1</b> or similarly simplified exact equiv
<b>9</b>	
<b>7 (i)</b> State $A = 42$	<b>B1</b>
State $k = \frac{1}{9}$	<b>B1</b> or 0.11 or greater accuracy
Attempt correct process for finding $m$	<b>M1</b> involving logarithms or equiv
Obtain $\frac{1}{9} \ln 2$ or 0.077	<b>A1</b> or 0.08 or greater accuracy
<b>4</b>	
<b>(ii)</b> Attempt solution for $t$ using either formula	<b>M1</b> using correct process (log's ms or T&I or ...)
Obtain 11.3	<b>A1</b> or greater accuracy; allow $11.3 \pm 0.1$
<b>2</b>	
<b>(iii)</b> Differentiate to obtain form $Be^{mt}$	<b>M1</b> where $B$ is different from $A$
Obtain $3.235e^{0.077t}$	<b>A1</b> or equiv; following their $A$ and $m$
Obtain 47.9	<b>A1</b> allow 48 or greater accuracy
<b>3</b>	

<p>8 (i) Show at least correct <math>\cos \theta \cos 60 + \sin \theta \sin 60</math> or <math>\cos \theta \cos 60 - \sin \theta \sin 60</math> Attempt expansion of both with exact numerical values attempted Obtain <math>\frac{1}{2}\sqrt{3} \sin \theta + \frac{5}{2} \cos \theta</math></p>	<p><b>B1</b> <b>M1</b> and with <math>\cos 60 \neq \sin 60</math> <b>A1</b> or exact equiv</p>
<p>(ii) Attempt correct process for finding <math>R</math> Attempt recognisable process for finding <math>\alpha</math> Obtain <math>\sqrt{7} \sin(\theta + 70.9)</math></p>	<p><b>M1</b> whether exact or approx <b>M1</b> allowing sin / cos muddles <b>A1</b> allow 2.65 for <math>R</math>; allow <math>70.9 \pm 0.1</math> for <math>\alpha</math></p>
<p>(iii) Attempt correct process to find any value of <math>\theta</math> + their <math>\alpha</math> Obtain any correct value for <math>\theta + 70.9</math> Attempt correct process to find <math>\theta</math> + their <math>\alpha</math> in 3rd quadrant Obtain 131 [SC for solutions with no working shown: Correct answer only B4; 131 with other answers B2]</p>	<p><b>M1</b> <b>A1</b> -158, -22, 202, 338, ... <b>M1</b> or several values including this <b>A1</b> or greater accuracy and no other</p>
<p>9 (i) Attempt use of quotient rule Obtain <math>\frac{75 - 15x^2}{(x^2 + 5)^2}</math>  Equate attempt at first derivative to zero and rearrange to solvable form Obtain <math>x = \sqrt{5}</math> or 2.24 Recognise range as values less than <math>y</math>-coord of st pt Obtain <math>0 \leq y \leq \frac{3}{2}\sqrt{5}</math></p>	<p><b>*M1</b> or equiv; allow <math>u / v</math> muddles <b>A1</b> or (unsimplified) equiv; this <b>M1A1</b>  available at any stage of question <b>M1</b> dep <b>*M</b> <b>A1</b> or greater accuracy <b>M1</b> allowing <math>&lt;</math> here <b>A1</b> any notation; with <math>\leq</math> now; any exact equiv</p>
<p>(ii) State <math>\sqrt{5}</math></p>	<p><b>B1</b> following their <math>x</math>-coord of st pt; condone answer <math>x \geq \sqrt{5}</math> but not inequality with <math>k</math></p>
<p>(iii) Equate attempt at first derivative to -1 and attempt simplification Obtain <math>x^4 - 5x^2 + 100 = 0</math> Attempt evaluation of discriminant or equiv Obtain -375 or equiv and conclude appropriately</p>	<p><b>*M1</b> and dependent on first <b>M</b> in part (i) <b>A1</b> or equiv involving 3 non-zero terms <b>M1</b> dep <b>*M</b> <b>A1</b></p>

# 4724 Core Mathematics 4

<p>1 (a) <math>2x^2 - 7x - 4 = (2x+1)(x-4)</math> or  <math>3x^2 + x - 2 = (3x-2)(x+1)</math></p> <p><math>\frac{2x+1}{3x-2}</math> as final answer; this answer only</p>	<p><b>B1</b></p> <p><b>B1</b> Do not ISW</p> <p><b>2</b></p>
<p>(b) For correct leading term <math>x</math> in quotient          For evidence of correct division process          Quotient = <math>x - 2</math></p> <p>Remainder = <math>x - 3</math></p>	<p><b>B1</b> <u>Identity method</u></p> <p><b>M1</b> <math>M1: x^3 + 2x^2 - 6x - 5 = Q(x^2 + 4x + 1) + R</math></p> <p><b>A1</b> <math>M1: Q = ax + b</math> or <math>x + b, R = cx + d</math> &amp; <math>\geq 2</math> ops          [N.B. If <math>Q = x + b</math>, this <math>\Rightarrow</math> 1 of the 2 ops]</p> <p><b>A1</b> <math>A2: a = 1, b = -2, c = 1, d = -3</math> SR: <u>B1</u> for two</p> <p><b>4</b></p>
<p>2 Parts with correct split of <math>u = \ln x, \frac{dv}{dx} = x^4</math></p> <p><math>\frac{x^5}{5} \ln x - \int \frac{x^5}{5} \cdot \frac{1}{x} (dx)</math></p> <p><math>\frac{x^5}{5} \ln x - \frac{x^5}{25}</math></p> <p>Correct method with the limits  <math>\frac{4e^5}{25} + \frac{1}{25}</math> ISW (Not '+c')</p>	<p><b>*M1</b> obtaining result <math>f(x) + /- \int g(x) dx</math></p> <p><b>A1</b></p> <p><b>A1</b></p> <p>dep<b>*M1</b> Decimals acceptable here</p> <p><b>A1</b> Accept equiv fract; like terms amalgamated</p> <p><b>5</b></p>
<p>3 (i) <math>\frac{d}{dx}(x^2 y) = x^2 \frac{dy}{dx} + 2xy</math> or <math>\frac{d}{dx}(xy^2) = 2xy \frac{dy}{dx} + y^2</math></p> <p>Attempt to solve their differentiated equation for <math>\frac{dy}{dx}</math></p> <p><math>\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}</math> only</p>	<p><b>*B1</b></p> <p>dep<b>*M1</b></p> <p><b>A1</b> WWW <b>AG</b> Must have intermediate line &amp; ...          ...could imply "=0" on 1<sup>st</sup> line</p> <p><b>3</b></p>
<p>(ii)(a) Attempt to solve <b>only</b> <math>y^2 - 2xy = 0</math> &amp; derive <math>y = 2x</math>          Clear indication why <math>y = 0</math> is not acceptable</p>	<p><b>B1</b> <b>AG</b> Any effort at solving <math>x^2 - 2xy = 0 \rightarrow B0</math></p> <p><b>B1</b> Substituting <math>y = 2x \rightarrow B0, B0</math></p> <p><b>2</b></p>
<p>(b) Attempt to solve <math>y = 2x</math> simult with <math>x^2 y - xy^2 = 2</math>          Produce <math>-2x^3 = 2</math> or <math>y^3 = -8</math>  <math>(-1, -2)</math> or <math>x = -1, y = -2</math> <b>only</b></p>	<p><b>M1</b></p> <p><b>A1</b> AEF</p> <p><b>A1</b></p> <p><b>3</b></p>



<b>4 (i)</b> For (either point) + $t$ (difference between vectors) $\mathbf{r} = (3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ or $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}) + t(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$ or $2\mathbf{i} - \mathbf{j} - \mathbf{k})$	<b>M1</b> <b>A1</b>	$'t'$ can be $'s'$ , $'\lambda'$ etc. $'\mathbf{r}'$ must be $'\mathbf{r}'$ but need not be bold Check other formats, e.g. $ta + (1-t)b$
<b>2</b>		
<b>(ii)</b> State/imply that their $\mathbf{r}$ and their $-2\mathbf{i} + \mathbf{j} + \mathbf{k}$ are perpendicular Consider scalar product = 0 Obtain $t = -\frac{1}{6}$ or $\frac{1}{6}$ or $-\frac{5}{6}$ or $\frac{5}{6}$ Subst their $t$ into their equation of $AB$ Obtain $\frac{1}{6}(16\mathbf{i} + 13\mathbf{j} + 19\mathbf{k})$ AEF	<b>*M1</b> N.B.This *M1 is dep on M1 being earned in (i) <b>dep*M1</b> <b>A1</b> <b>M1</b> <b>A1</b> Accept decimals if clear	
<b>5</b>		
<b>5 (i)</b> $(1-x)^{\frac{1}{2}} = 1 - \frac{1}{2}x - \frac{1}{8}x^2$ ignoring $x^3$ etc $(1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2$ ignoring $x^3$ etc Product = $1 - x + \frac{1}{2}x^2$ ignoring $x^3$ etc	<b>B2</b> SR Allow B1 for $1 - \frac{1}{2}x + kx^2$ , $k \neq -\frac{1}{8}$ or 0 <b>B2</b> SR Allow B1 for $1 - \frac{1}{2}x + kx^2$ , $k \neq \frac{3}{8}$ or 0 <b>B1</b> AG; with (at least) 1 intermediate step (cf $x^2$ )	
<b>5</b>		
<b>(ii)</b> $\frac{\sqrt{5}}{9}$ or $\frac{\sqrt{5}}{3}$ seen $\frac{37}{49}$ or $1 - \frac{2}{7} + \frac{1}{2}\left(\frac{2}{7}\right)^2$ seen $\frac{\sqrt{5}}{3} \approx \frac{37}{49} \Rightarrow \sqrt{5} \approx \frac{111}{49}$	<b>B1</b> <b>B1</b> <b>B1</b> AG	
<b>3</b>		
<b>6 (i)</b> Produce at least 2 of the 3 relevant equations in $t$ and $s$ Solve for $t$ and $s$ $(t, s) = (4, -3)$ AEF Subst $(4, -3)$ into suitable equation(s) & show consistency	<b>M1</b> $1 + 2t = 12 + s$ , $3t = -4s$ , $-5 + 4t = 5 - 2s$ <b>M1</b> <b>*A1</b> <b>dep*A1</b> Either into "3 <sup>rd</sup> " eqn or into all 3 coordinates. N.B. Intersection coords not asked for	
<b>4</b>		
<b>(ii)</b> Method for finding magnitude of any vector Method for finding scalar product of any 2 vectors Using $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{ \mathbf{a}   \mathbf{b} }$ AEF for the correct 2 vectors 137 (136.8359) or 43.2(43.164...)	<b>*M1</b> Expect $\sqrt{29}$ and $\sqrt{21}$ <b>*M1</b> Expect $-18$ <b>dep*M1</b> Should be $-\frac{18}{\sqrt{29}\sqrt{21}}$ <b>A1</b> 2.39 (2.388236..) or 0.753(0.75335...) rads	
<b>4</b>		

7 (i) Correct (calc) method for dealing with $\frac{1}{\sin x}$ or $(\sin x)^{-1}$	<b>M1</b>	
Obtain $-\frac{\cos x}{\sin^2 x}$ or $-(\sin x)^{-2} \cos x$	<b>A1</b>	
Show manipulation to $-\operatorname{cosec} x \cot x$ (or vice-versa)	<b>A1</b>	WWW <b>AG</b> with $\geq 1$ line intermed working
<b>3</b>		
(ii) Separate variables, $\int (-)\frac{1}{\sin x \tan x} dx = \int \cot t dt$	<b>M1</b>	or $\int \frac{1}{\sin x \tan x} dx = \int (-)\cot t dt$
Style: For the M1 to be awarded, dx and dt must appear on correct sides or there must be $\int$ sign on both sides		
$\int -\operatorname{cosec} x \cot x dx = \operatorname{cosec} x (+c)$	<b>A1</b>	or $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x$
$\int \cot t dt = \ln  \sin t $ or $\ln  \sin t  (+c)$	<b>B1</b>	or $\int -\cot t dt = -\ln  \sin t $ or $-\ln  \sin t $
Subst $(t, x) = \left(\frac{1}{2}\pi, \frac{1}{6}\pi\right)$ into their equation containing 'c'	<b>M1</b>	and attempt to find 'c'
$\operatorname{cosec} x = \ln  \sin t  + 2$ or $\ln  \sin t  + 2$	<b>A1</b>	WWW ISW; $\operatorname{cosec} \frac{\pi}{6}$ to be changed to 2
<b>5</b>		
8 (i) $A(t+1) + B = 2t$ $A = 2$ $B = -2$	<b>M1</b> <b>A1</b> <b>A1</b>	<b>Beware:</b> correct values for A and/or B can be ... ... obtained from a wrong identity <b>Alt method:</b> subst suitable values into given... ...expressions
<b>3</b>		
(ii) Attempt to connect dx and dt $dx = t dt$ s.o.i. AEF	<b>M1</b> <b>A1</b>	But not just $dx = dt$ . As <b>AG</b> , look carefully.
$x + \sqrt{2x-1} \rightarrow \frac{t^2+1}{2} + t = \frac{(t+1)^2}{2}$ s.o.i.	<b>B1</b>	Any wrong working invalidates
$\int \frac{2t}{(t+1)^2} dt$	<b>A1</b>	<b>AG</b> WWW The 'dt' must be present
<b>4</b>		
(iii) $\int \frac{1}{t+1} dt = \ln(t+1)$	<b>B1</b>	Or parts $u = 2t, dv = (t+1)^{-2}$ or subst $u = t+1$
$\int \frac{1}{(t+1)^2} dt = -\frac{1}{t+1}$	<b>B1</b>	
Attempt to change limits (expect 1 & 3) and use f(t)	<b>M1</b>	or re-substitute and use 1 and 5 on g(x)
$\ln 4 - \frac{1}{2}$	<b>A1</b>	AEF (like terms amalgamated); if A0 A0 in (i), then final A0
<b>4</b>		

- 9 (i)  $A: \theta = \frac{1}{2}\pi$  (accept  $90^\circ$ ) **B1**  
 $B: \theta = 2\pi$  (accept  $360^\circ$ ) **B2** SR If B0 awarded for point B, allow B1 SR for any angle s.t.  $\sin \theta = 0$

3
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- (ii)  $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$  **M1** or  $\frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$  Must be used, not just quoted

$$\frac{dx}{d\theta} = 2 + 2 \cos 2\theta$$

**B1**

$$2 + 2 \cos 2\theta = 4 \cos^2 \theta \text{ with } \geq 1 \text{ line intermed work}$$

**\*B1**

$$\frac{dy}{dx} = \frac{4 \cos \theta}{2 + 2 \cos 2\theta} \quad \text{s.o.i.}$$

**A1** This & previous line are interchangeable

$$= \sec \theta$$

dep**\*A1** WWW **AG**

5
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- (iii) Equating  $\sec \theta$  to 2 and producing at least one value of  $\theta$  **M1** degrees or radians

$$(x =) -\frac{2}{3}\pi - \frac{\sqrt{3}}{2}$$

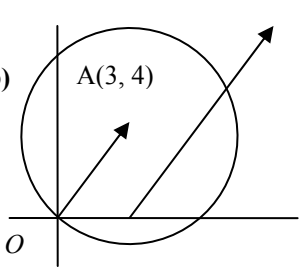
**A1** 'Exact' form required

$$(y =) -2\sqrt{3}$$

**A1** 'Exact' form required

3
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# 4725 Further Pure Mathematics 1

<p>1 (i) <math>\begin{pmatrix} 1 &amp; 1 \\ 5 &amp; -1 \end{pmatrix}</math></p>	<p>B1 Two elements correct B1 All four elements correct <b>2</b></p>
<p>(ii) EITHER</p> $\frac{1}{3} \begin{pmatrix} 2 & -1 \\ -5 & 4 \end{pmatrix}$ <p>OR</p>	<p>B1 Both diagonals correct B1 Divide by determinant <b>2</b> B1 Solve sim. eqns. 1<sup>st</sup> column correct B1 2<sup>nd</sup> column correct</p>
<p>2 (i) 5 0.927 or 53.1°</p>	<p>B1 Correct modulus B1 Correct argument, any equivalent form <b>2</b></p>
<p>(ii)(a)</p> <p>(b)</p> 	<p>B1 Circle centre A (3, 4) B1 Through O, allow if centre is (4, 3) <b>2</b> B1 Half line with +ve slope B1 Starting at (3, 0) B1 Parallel to OA, (implied by correct arg shown) <b>3</b></p>
<p>3 (i) <math>\frac{r}{(r+1)!}</math></p>	<p>M1 Common denominator of <math>(r+1)!</math> or <math>r!(r+1)!</math> A1 Obtain given answer correctly <b>2</b></p>
<p>(ii) <math>1 - \frac{1}{(n+1)!}</math></p>	<p>M1 Express terms as differences using (i) A1 At least 1<sup>st</sup> two and last term correct M1 Show pairs cancelling A1 Correct answer a.e.f. <b>4</b></p>
<p>4</p>	<p>B1 Establish result is true, for <math>n = 1</math> ( or 2 or 3 ) M1 Attempt to multiply A and A<sup>n</sup>, or vice versa M1 Correct process for matrix multiplication A1 Obtain <math>3^{n+1}</math>, 0 and 1 A1 Obtain <math>\frac{1}{2}(3^{n+1} - 1)</math> A1 Statement of Induction conclusion, only if 5 marks earned, but may be in body of working <b>6</b></p>

5		M1	Express as difference of two series
		M1	Use standard results
	$\frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1)$	A1	Correct unsimplified answer
		M1	Attempt to factorise
		A1	At least factor of $n(n+1)$
	$\frac{1}{12}n(n+1)(3n+2)(n-1)$	A1	Obtain correct answer
			<b>6</b>
6 (i)	$3 - i$	B1	Conjugate stated
			<b>1</b>
(ii)	<i>EITHER</i>	M1	Use sum of roots
		A1	Obtain correct answer
		M1	Use sum of pairs of roots
		A1	Obtain correct answer
		M1	Use product of roots
	$a = -8, b = 22, c = -20$	A1	Obtain correct answers
			<b>6</b>
	<i>OR</i>	M1	Attempt to find a quadratic factor
		A1	Obtain correct factor
	$a = -8, b = 22, c = -20$	M1	Expand linear and quadratic factors
	<i>OR</i>	A1A1A1	Obtain correct answers
		M1	Substitute 1 imaginary & the real root into eqn
		M1	Equate real and imaginary parts
		M1	Attempt to solve 3 eqns.
	$a = -8, b = 22, c = -20$	A1A1A1	Obtain correct answers
7 (i)		B1	Enlargement (centre $O$ ) scale factor 6
			<b>1</b>
(ii)		B1	Reflection
		B1	Mirror line is $y = x$
			<b>2</b>
(iii)		B1	Stretch in $y$ direction
		B1	Scale factor 6, must be a stretch
			<b>2</b>
(iv)		B1	Rotation
		B1	$36.9^\circ$ clockwise or equivalent
			<b>2</b>

8	$\alpha + \beta = -k$ $\alpha\beta = 2k$	<b>B1</b> State or use correct value <b>B1</b> State or use correct value <b>M1</b> Attempt to express sum of new roots in terms of $\alpha + \beta$ , $\alpha\beta$ <b>A1</b> Obtain correct expression <b>A1</b> Obtain correct answer a.e.f. <b>B1</b> Correct product of new roots seen <b>B1ft</b> Obtain correct answer, must be an eqn. <div style="border: 1px solid black; display: inline-block; padding: 2px;">7</div>	
	$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$ $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{1}{2}(k - 4)$ $\alpha'\beta' = 1$ $x^2 - \frac{1}{2}(k - 4)x + 1 = 0$	<p>Alternative for last 5 marks</p> <b>M1</b> Obtain expression for $u = \frac{\alpha}{\beta}$ in terms of $k$ and $\alpha$ or $k$ and $\beta$ <b>A1</b> Obtain a correct expression <b>A1</b> rearrange to get $\alpha$ in terms of $u$ <b>M1</b> Substitute into given equation <b>A1</b> Obtain correct answer	
9	(i)	<b>M1</b> Attempt to equate real and imaginary parts of $(x + iy)^2$ and $5 + 12i$ <b>A1</b> Obtain both results <b>M1</b> Eliminate to obtain a quadratic in $x^2$ or $y^2$ <b>M1</b> Solve a 3 term quadratic & obtain $x$ or $y$ <b>A1</b> Obtain correct answers as complex nos. <div style="border: 1px solid black; display: inline-block; padding: 2px;">5</div>	
	(ii)	$x^2 - y^2 = 5$ and $xy = 6$  $\pm(3 + 2i)$	<b>B1B1</b> Correct real and imaginary parts <div style="border: 1px solid black; display: inline-block; padding: 2px;">2</div>
	(iii)	$x^2 = 5 \pm 12i$ $x = \pm(3 \pm 2i)$	<b>M1</b> Attempt to solve a quadratic equation <b>A1</b> Obtain correct answers <b>A1A1</b> Each pair of correct answers a.e.f. <div style="border: 1px solid black; display: inline-block; padding: 2px;">4</div>

10 (i)

**M1** Find value of det **AB****A1** Correct value 2 seen**2**

(ii)

**M1** Show correct process for adjoint entries**A1** Obtain at least 4 correct entries in adjoint**B1** Divide by their determinant

$$(\mathbf{AB})^{-1} = \frac{1}{2} \begin{pmatrix} 0 & 3 & -1 \\ 0 & -1 & 1 \\ 2 & 6-3a & a-6 \end{pmatrix}$$

**A1** Obtain completely correct answer**4**

(iii) EITHER

**M1** State or imply  $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ **A1** Obtain  $\mathbf{B}^{-1} = (\mathbf{AB})^{-1} \times \mathbf{A}$ **M1** Correct multiplication process seen**A1** Obtain three correct elements


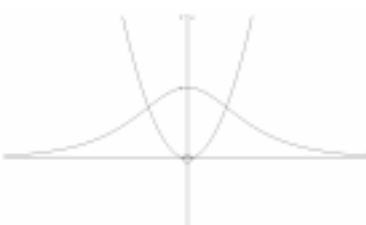
$$\mathbf{B}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ -6 & 2 & -2 \end{pmatrix}$$

**A1** All elements correct**5**

OR

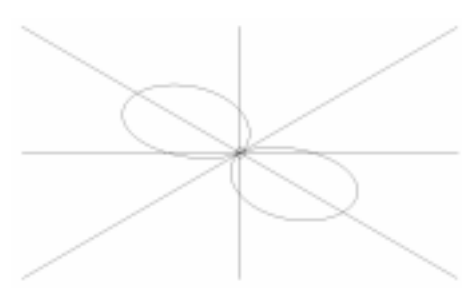
**M1** Attempt to find elements of **B****A1** All correct**M1** Correct process for  $\mathbf{B}^{-1}$ **A1** 3 elements correct**A1** All elements correct

## 4726 Further Pure Mathematics 2

<p>1 Write as <math>\frac{A}{x-2a} + \frac{Bx+C}{x^2+a^2}</math>            Get <math>2ax = A(x^2+a^2) + (Bx+C)(x-2a)</math>            Choose values of <math>x</math> and/or equate coeff.            Get <math>A = \frac{4}{5}, B = -\frac{4}{5}, C = \frac{2}{5}a</math></p>	<p>M1 Accept <math>C=0</math>            A1√ Follow-on for <math>C=0</math>            M1 Must lead to at least one of their <math>A, B, C</math>            A1 For two correct from correct working only            A1 For third correct  <b>5</b></p>
<p>2</p> 	<p>B1 Get <math>(4,0), (3,0), (-2,0)</math> only            B1 Get <math>(0, \sqrt{5})</math> as “maximum”</p> <hr/> <p>B1 Meets <math>x</math>-axis at <math>90^\circ</math> at all crossing points            B1 Use <math>-2 \leq x \leq 3</math> and <math>x \geq 4</math> only            B1 Symmetry in <math>Ox</math>  <b>5</b></p>
<p>3 Quote/derive <math>dx = \frac{2}{1+t^2} dt</math>            Replace all <math>x</math> and <math>dx</math> from their expressions            Tidy to <math>2/(3t^2+1)</math>            Get <math>k \tan^{-1}(At)</math>            Get <math>k = \frac{2}{3}\sqrt{3}, A = \sqrt{3}</math>            Use limits correctly to <math>\frac{2}{9}, \sqrt{3}\pi</math></p>	<p>B1</p> <p>M1 Not <math>dx=dt</math>; ignore limits            A1 Not <math>a/(3t^2+1)</math>            M1 Allow <math>A=1</math> if from <math>p/(t^2+1)</math> only            A1√ Allow <math>k=a/\sqrt{3}</math> from line 3; AEEF            A1 AEEF  <b>6</b></p>
<p>4 (i)</p> 	<p>B1 Correct <math>y = x^2</math></p> <hr/> <p>B1 Correct shape/asymptote            B1 Crossing <math>(0,1)</math>  <b>3</b></p>
<p>(ii) Define <math>\operatorname{sech} x = 2/(e^x + e^{-x})</math>            Equate their expression to <math>x^2</math> and attempt to simplify            Clearly get A.G.</p>	<p>B1 AEEF            M1            A1  <b>3</b></p>
<p>(iii) Cobweb            Values <math>&gt;</math> and then <math>&lt;</math> root</p>	<p>B1            B1 Only from cobweb  <b>2</b></p>

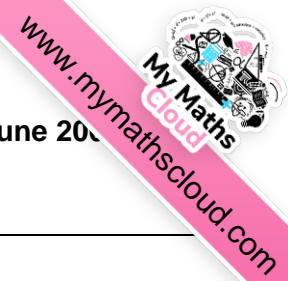


<b>5 (i)</b> Factorise to $\tan^{n-2}x(1+\tan^2x)$ Clearly use $1+\tan^2 = \sec^2$ Integrate to $\tan^{n-1}x/(n-1)$ Use limits and tidy to A.G.	<b>B1</b> Or use $\tan^n x = \tan^{n-2} x \cdot \tan^2 x$ <b>M1</b> Allow wrong sign <b>A1</b> Quote or via substitution <b>A1</b> Must be clearly derived <b>4</b>
<b>(ii)</b> Get $3(I_4 + I_2) = 1, I_2 + I_0 = 1$  Attempt to evaluate $I_0$ (or $I_2$ ) Get $\frac{1}{4}\pi$ (or $1 - \frac{1}{4}\pi$ ) Replace to $\frac{1}{4}\pi - \frac{2}{3}$	<b>B1</b> Write down one correct from reduction formula <b>M1</b> $I_2 = a \tan x + b, a, b \neq 0$ <b>A1</b> <b>A1</b> <b>4</b>
<b>6 (i)</b> Attempt to use N-R of correct form with clear $f'(x)$ used Get 2.633929, 2.645672	<b>M1</b> <b>A1</b> For one correct to minimum of 6 d.p. <b>A1</b> √ For other correct from their $x_2$ in correct NR <b>3</b>
<b>(ii)</b> $\sqrt{7}$	<b>B1</b> Allow $\pm$ <b>1</b>
<b>(iii)</b> Get $e_1 = 0.14575, e_2 = 0.01182$ Get $e_3 = 0.00008$ Verify both $\approx 0.00008$	<b>B1</b> √ From their values <b>B1</b> √ <b>B1</b> From $0.000077\dots$ or $0.01182^3/0.14575^2$ <b>3</b>
<b>7 (i)</b> Attempt quotient/product on bracket Get $-3/(2+x)^2$ Use Formulae Booklet or derive from $\tanh y = (1-x)/(2+x)$ Get $\frac{-3}{(2+x)^2} \cdot \frac{1}{1 - ((1-x)/(2+x))^2}$ Clearly tidy to A.G. Get $f''(x) = 2/(1+2x)^2$	<b>M1</b> <b>A1</b> May be implied <b>M1</b> Attempt $\tanh^{-1}$ part in terms of $x$ <b>A1</b> √ From their results above <b>A1</b> <b>B1</b> cao <b>6</b>
<b>(ii)</b> Attempt $f(0), f'(0)$ and $f''(0)$ Get $\tanh^{-1} \frac{1}{2}, -1$ and $2$ Replace $\tanh^{-1} \frac{1}{2} = \frac{1}{2} \ln 3 (= \ln \sqrt{3})$ Get $\ln \sqrt{3} - x + x^2$	<b>SC</b> Use reasonable $\ln$ definition <b>M1</b> Get $y = \frac{1}{2} \ln((1-k)/(1+k))$ for $k = (1-x)/(1+2x)$ <b>A1</b> Tidy to $y = \frac{1}{2} \ln(3/(1+2x))$ <b>A1</b> Attempt chain rule <b>M1</b> Clearly tidy to A.G. <b>A1</b> Get $f''(x)$ <b>B1</b> <b>M1</b> From their differentiation <b>A1</b> √ <b>B1</b> Only <b>A1</b> <b>4</b> <b>SC</b> Use standard expansion from $\frac{1}{2} \ln 3 - \frac{1}{2} \ln(1+2x)$

<b>8 (i)</b> Attempt to solve $r = 0$ Get $\alpha = \frac{1}{4}\pi$	<b>M1</b> <b>A1</b> From correct method; ignore others; allow $\theta$ <b>2</b>
<b>(ii) (a)</b> Get $1 - \sin((2k+1)\pi - 2\theta)$ Expand as $\sin(A+B)$ Use $k$ as integer so $\sin(2k+1)\pi = 0$ , And $\cos(2k+1)\pi = -1$	<b>M1</b> Attempt $f(\frac{1}{2}(2k+1)\pi - \theta)$ , leading to $2\theta$ here <b>M1</b> Or discuss periodicity for general $k$ <b>A1</b> Needs a clear explanation <b>3</b>
<b>(b)</b> Quote $\frac{1}{4}(2k+1)\pi$  Select or give $k = 0, 1, 2, 3$	<b>B1</b> For general answer or 2 correct (ignore other answers given) <b>B1</b> For all 4 correct in $0 \leq \theta < 2\pi$ <b>2</b>
<b>(iii)</b> roughly  	<b>B1</b> Correct shape; 2 branches only, as shown  <b>B1</b> Clear symmetry in correct rays <b>B1</b> Get max. $r = 2$ <b>B1</b> At $\theta = \frac{3}{4}\pi$ and $\frac{7}{4}\pi$ ; both required (allow correct answers not in $0 \leq \theta < 2\pi$ here) <b>4</b>
<b>9 (i)</b> Attempt to use parts Divide out $x/(1+x)$ Correct answer $x \ln(1+x) - x + \ln(1+x)$ Limits to correct A.G.	<b>M1</b> Two terms, one yet to be integrated <b>M1</b> Or use substitution <b>A1</b> <b>A1</b> <b>4</b> <b>SC</b> Quote $\int \ln x \, dx$ <b>M1</b> Clear use of limits to A.G. <b>A1</b> <b>SC</b> Attempt to differentiate by product rule <b>M1</b> Clear use of limits to A.G. <b>A1</b>
<b>(ii) (a)</b> Use sum of areas of rect. Area under curve (between limits 0 and 70) Areas = $1 \times \text{heights} = 1(\ln 2 + \ln 3 + \dots + \ln 70)$	<b>B1</b> <b>B1</b> <u>Areas</u> to be specified <b>2</b>
<b>(b)</b> Explain use of 69 Explain first rectangle Areas as above > area under curve	<b>B1</b> Allow diagram or use of left shift of 1 unit <b>B1</b> <b>B1</b> <b>3</b>
<b>(c)</b> Show/quote $\ln 2 + \ln 3 + \dots + \ln 70 = \ln 70!$ Use $N = 69, 70$ in (i)  Get 228.3, 232.7	<b>B1</b> <b>M1</b> No other numbers; may be implied by 228.39.. or 232.65.. seen; allow 228.4, 232.6 or 232.7 <b>A1</b> <b>3</b>

# 4727 Further Pure Mathematics 3

1 (a)(i)	$e, r^3, r^6, r^9$	M1	For stating $e, r^m$ (any $m \geq 2$ ), and 2 other different elements in terms of $e$ and $r$
		A1 2	For all elements correct
(ii)	$r$ generates $G$	B1 1	For this or any statement equivalent to: all elements of $G$ are included in a group with $e$ and $r$ OR order of $r >$ order of all possible proper subgroups
(b)	$m, n, p, mn, np, pm$	B1	For any 3 orders correct
		B1 2	For all 6 correct and no extras (Ignore 1 and $mnp$ )
<b>5</b>			
2	METHOD 1		
	$[1, 3, 2] \times [1, 2, -1]$	M1	For attempt to find normal vector, e.g. by finding vector product of correct vectors, or Cartesian equation
	$\mathbf{n} = k[-7, 3, -1]$ OR $7x - 3y + z = c$ ( $= 17$ )	A1	For correct vector OR LHS of equation
	$\theta = \sin^{-1} \frac{ [1, 4, -1] \cdot [-7, 3, -1] }{\sqrt{1^2 + 4^2 + 1^2} \sqrt{7^2 + 3^2 + 1^2}}$	M1√	For using correct vectors for line and plane f.t. from normal
		M1*	For using scalar product of line and plane vectors
		M1	For calculating both moduli in denominator
	$\theta = \sin^{-1} \frac{6}{\sqrt{18}\sqrt{59}} = 10.6^\circ$	A1√	For scalar product. f.t. from their numerator
	(10.609...°, 0.18517...)	(*dep) A1 7	For correct angle
	METHOD 2		
	$[1, 3, 2] \times [1, 2, -1]$	M1	For attempt to find normal vector, e.g. by finding vector product of correct vectors, or Cartesian equation
	$\mathbf{n} = k[-7, 3, -1]$ OR $7x - 3y + z = c$	A1	For correct vector OR LHS of equation
	$7x - 3y + z = 17$	M1√	For attempting to find RHS of equation f.t. from $\mathbf{n}$ or LHS of equation
	$d = \frac{ 21 - 12 + 2 - 17 }{\sqrt{7^2 + 3^2 + 1^2}} = \frac{6}{\sqrt{59}}$	M1	For using distance formula from a point on the line, e.g.
		A1√	(3, 4, 2), to the plane For correct distance. f.t. from equation
	$\theta = \sin^{-1} \frac{\frac{6}{\sqrt{59}}}{\sqrt{1^2 + 4^2 + 1^2}} = 10.6^\circ$	M1	For using trigonometry
	(10.609...°, 0.18517...)	A1	For correct angle
<b>7</b>			
3 (i)	$\frac{dz}{dx} = 1 + \frac{dy}{dx}$	M1	For differentiating substitution (seen or implied)
	$\frac{dz}{dx} - 1 = \frac{z+3}{z-1} \Rightarrow \frac{dz}{dx} = \frac{2z+2}{z-1} = \frac{2(z+1)}{z-1}$	A1	For correct equation in $z$ AEF
		A1 3	For correct simplification to AG
(ii)	$\int \frac{z-1}{z+1} dz = 2 \int dx$	B1	For $\int \frac{z-1}{z+1} (dz)$ and $\int (1) (dx)$ seen or implied
	$\Rightarrow \int 1 - \frac{2}{z+1} dz$ OR $\int 1 - \frac{2}{u} du = 2x (+c)$	M1	For rearrangement of LHS into integrable form OR substitution e.g. $u = z+1$ or $u = z-1$
	$\Rightarrow z - 2 \ln(z+1)$ OR $z+1 - 2 \ln(z+1)$	A1	For correct integration of LHS as $f(z)$
	$= 2x (+c)$		
	$\Rightarrow -2 \ln(x+y+1) = x - y + c$	A1 4	For correct general solution AEF



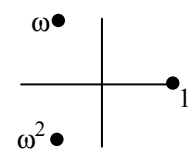
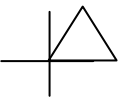
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4 (i)	$\cos^5 \theta = \left( \frac{e^{i\theta} + e^{-i\theta}}{2} \right)^5$	B1	For $\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$ seen or implied $z$ may be used for $e^{i\theta}$ throughout
	$\cos^5 \theta = \frac{1}{32} (e^{i\theta} + e^{-i\theta})^5$	M1	For expanding $(e^{i\theta} + e^{-i\theta})^5$ . At least 3 terms and 2 binomial coefficients required <i>OR</i> reasonable attempt at expansion in stages
	$\cos^5 \theta = \frac{1}{32} (e^{5i\theta} + e^{-5i\theta} + 5(e^{3i\theta} + e^{-3i\theta}) + 10(e^{i\theta} + e^{-i\theta}))$	A1	For correct binomial expansion
	$\cos^5 \theta = \frac{1}{16} (\cos 5\theta + 5 \cos 3\theta + 10 \cos \theta)$	M1 A1	For grouping terms and using multiple angles For answer obtained correctly <b>AG</b>
<hr style="border-top: 1px dashed black;"/>			
(ii)	$\cos \theta = 16 \cos^5 \theta$	B1	For stating correct equation of degree 5 <i>OR</i> $1 = 16 \cos^4 \theta$ <b>AEF</b>
	$\Rightarrow \cos \theta = 0, \cos \theta = \pm \frac{1}{2}$	M1	For obtaining at least one of the values of $\cos \theta$ from $\cos \theta = k \cos^5 \theta$ <i>OR</i> from $1 = k \cos^4 \theta$
	$\Rightarrow \theta = \frac{1}{2}\pi, \frac{1}{3}\pi, \frac{2}{3}\pi$	A1 A1	A1 for any two correct values of $\theta$ A1 <b>4</b> A1 for the 3rd value and no more in $0, \theta, \pi$ Ignore values outside $0, \theta, \pi$

<b>5 (i)</b>	<b>METHOD 1</b>		
	Lines meet where		
	$(x =) k + 2\lambda = k + \mu$	M1	For using parametric form to find where lines meet
	$(y =) -1 - 5\lambda = -4 - 4\mu$	A1	For at least 2 correct equations
	$(z =) 1 - 3\lambda = -2\mu$		
	$\Rightarrow \lambda = -1, \mu = -2$	M1	For attempting to solve any 2 equations
		A1	For correct values of $\lambda$ and $\mu$
		B1	For attempting a check in 3rd equation
	$\Rightarrow (k - 2, 4, 4)$	A1	OR verifying point of intersection is on both lines
		A1	6 For correct point of intersection (allow vector)
			<b>SR</b> For finding $\lambda$ OR $\mu$ and point of intersection, but no check, award up to M1 A1 M1 A0 B0 A1
	<b>METHOD 2</b>		
	$d = \frac{ [0, 3, 1] \cdot [2, -5, -3] \times [1, -4, -2] }{ \mathbf{b} \times \mathbf{c} }$		For using $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ with appropriate vectors (division by $ \mathbf{b} \times \mathbf{c} $ is not essential)
	$d = c[0, 3, 1] \cdot [-2, 1, -3] = 0$	B1	and showing $d = 0$ correctly
	$\Rightarrow$ lines intersect		
	Lines meet where		
	$(x =) (k +) 2\lambda = (k +) \mu$	M1	For using parametric form to find where lines meet
	$(y =) -1 - 5\lambda = -4 - 4\mu$	A1	For at least 2 correct equations
	$(z =) 1 - 3\lambda = -2\mu$		
	$\Rightarrow \lambda = -1, \mu = -2$	M1	For attempting to solve any 2 equations
		A1	For correct value of $\lambda$ OR $\mu$
	$\Rightarrow (k - 2, 4, 4)$	A1	For correct point of intersection (allow vector)
	<b>METHOD 3</b>		
	e.g. $x - k = \frac{2(y+1)}{-5} = \frac{y+4}{-4}$	M1	For solving one pair of simultaneous equations
	$\Rightarrow y = 4$	A1	For correct value of $x, y$ or $z$
	$\frac{z-1}{-3} = \frac{y+1}{-5}$	M1	For solving for the third variable
	$x = k - 2$ OR $z = 4$	A1	For correct values of 2 of $x, y$ and $z$
	$x - k = \frac{z}{-2}$ checks with $x = k - 2, z = 4$	B1	For attempting a check in 3rd equation
	$\Rightarrow (k - 2, 4, 4)$	A1	For correct point of intersection (allow vector)
<b>(ii)</b>	<b>METHOD 1</b>		
	$\mathbf{n} = [2, -5, -3] \times [1, -4, -2]$	M1	For finding vector product of 2 directions
	$\mathbf{n} = c[-2, 1, -3]$	A1	For correct normal
	$(1, -1, 1)$ OR $(1, -4, 0)$ OR $(-1, 4, 4)$	M1	<b>SR</b> Following Method 2 for (i), award M1 A1√ for $\mathbf{n}$ , f.t. from their $\mathbf{n}$
	$\Rightarrow 2x - y + 3z = 6$	A1	4 For substituting a point in LHS
			For correct equation of plane <b>AEF cartesian</b>
	<b>METHOD 2</b>		
	$\mathbf{r} = [1, -1, 1] + \lambda[2, -5, -3] + \mu[1, -4, -2]$	M1	For using vector equation of plane (OR $[1, -4, 0]$ for <b>a</b> )
	$x = 1 + 2\lambda + \mu$		
	$y = -1 - 5\lambda - 4\mu$	A1	For writing 3 linear equations
	$z = 1 - 3\lambda - 2\mu$		
	$\Rightarrow 2x - y + 3z = 6$	M1	For eliminating $\lambda$ and $\mu$
		A1	For correct equation of plane <b>AEF cartesian</b>

6 (i)	When $a, b$ have opposite signs, $a b  = \pm ab, b a  = \mp ba \Rightarrow a b  \neq b a $	M1	For considering sign of $a b $ OR $b a $ in general or in a specific case
		A1	For showing that $a b  \neq b a $
			Note that $ x  = \sqrt{x^2}$ may be used
(ii)	$(a \circ b) \circ c = (a b ) \circ c = a b  c $ OR $a bc $	M1	For using 3 distinct elements and simplifying $(a \circ b) \circ c$ OR $a \circ (b \circ c)$
		A1	For obtaining correct answer
	$a \circ (b \circ c) = a \circ (b c ) = a b c  = a b  c $ OR $a bc $	M1	For simplifying the other bracketed expression
		A1	For obtaining the same answer
(iii)	EITHER $a \circ e = a e  = a \Rightarrow e = \pm 1$	B1*	For stating $e = \pm 1$ OR no identity
		M1	For attempting algebraic justification of +1 and -1 for $e$
	OR $e \circ a = e a  = a$	A1	For deducing no (unique) identity
	$\Rightarrow e = 1$ for $a > 0, e = -1$ for $a < 0$		
	Not a group	B1	For stating not a group
		(*dep)	
		4	
		10	

7 (i)		B1 1	<div style="border: 1px solid black; padding: 2px; margin-bottom: 5px;">Polar or cartesian values of <math>\omega</math> and <math>\omega^2</math> may be used anywhere in this question</div> For showing 3 points in approximately correct positions
Allow $\omega$ and $\omega^2$ interchanged, or unlabelled			
(ii)	<p><i>EITHER</i> <math>1 + \omega + \omega^2</math> = sum of roots of cubic = 0</p> <p><i>OR</i> <math>\omega^3 = 1 \Rightarrow (\omega - 1)(\omega^2 + \omega + 1) = 0</math> <math>\Rightarrow 1 + \omega + \omega^2 = 0</math> (for <math>\omega \neq 1</math>)</p> <p><i>OR</i> sum of G.P. <math>1 + \omega + \omega^2 = \frac{1 - \omega^3}{1 - \omega} \left( = \frac{0}{1 - \omega} \right) = 0</math></p>	M1 A1 2	For result shown by any correct method <b>AG</b>
<i>OR</i>	 shown on Argand diagram or explained in terms of vectors	Reference to vectors in part (i) diagram may be made	
<i>OR</i>	$1 + \text{cis } \frac{2}{3}\pi + \text{cis } \frac{4}{3}\pi = 1 + \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) + \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 0$		
(iii) (a)	$(2 + \omega)(2 + \omega^2) = 4 + 2(\omega + \omega^2) + \omega^3$ = 4 - 2 + 1 = 3	M1 A1 2	For using $1 + \omega + \omega^2 = 0$ <i>OR</i> values of $\omega$ , $\omega^2$ For correct answer
(b)	$\frac{1}{2 + \omega} + \frac{1}{2 + \omega^2} = \frac{2 + (\omega + \omega^2) + 2}{3} = 1$	M1 A1√ 2	For combining fractions <i>OR</i> multiplying top and bottom of 2 fractions by complex conjugates For correct answer f.t. from (a)
(iv)	For the cubic $x^3 + px^2 + qx + r = 0$		
METHOD 1			
	$\sum \alpha = 2 + 1 = 3 \Rightarrow p = -3$	M1	For calculating two of $\sum \alpha$ , $\sum \alpha\beta$ , $\alpha\beta\gamma$
	$\sum \alpha\beta = \frac{2}{2 + \omega} + \frac{2}{2 + \omega^2} + \frac{1}{3} = \frac{7}{3} (= q)$	M1	For calculating all of $\sum \alpha$ , $\sum \alpha\beta$ , $\alpha\beta\gamma$ <i>OR</i> all of $p$ , $q$ , $r$
	$\alpha\beta\gamma = \frac{2}{3} \left( \Rightarrow r = -\frac{2}{3} \right)$	A1	For at least two of $\sum \alpha$ , $\sum \alpha\beta$ , $\alpha\beta\gamma$ correct (or values of $p$ , $q$ , $r$ )
	$\Rightarrow 3x^3 - 9x^2 + 7x - 2 = 0$	A1 4	For correct equation <b>CAO</b>
METHOD 2			
	$\left(x - 2\right)\left(x - \frac{1}{2 + \omega}\right)\left(x - \frac{1}{2 + \omega^2}\right) = 0$		
	$x^3 + \left(-2 - \frac{1}{2 + \omega} - \frac{1}{2 + \omega^2}\right)x^2$	M1	For multiplying out LHS in terms of $\omega$ or $\text{cis } \frac{1}{3}k\pi$
	$+ \left(\frac{1}{(2 + \omega)(2 + \omega^2)} + \frac{2}{2 + \omega} + \frac{2}{2 + \omega^2}\right)x$		
	$-\frac{2}{(2 + \omega)(2 + \omega^2)} = 0$	M1	For simplifying, using parts (ii), (iii) or values of $\omega$
	$\Rightarrow x^3 - 3x^2 + \frac{7}{3}x - \frac{2}{3} = 0$	A1	For at least two of $p$ , $q$ , $r$ correct
	$\Rightarrow 3x^3 - 9x^2 + 7x - 2 = 0$	A1	For correct equation <b>CAO</b>



8 (i)	$m^2 + 1 = 0 \Rightarrow m = \pm i$	M1	For stating and attempting to solve correct auxiliary equation
	$\Rightarrow$ C.F.	A1	For correct C.F. (must be in trig form)
	$(y =) Ce^{ix} + De^{-ix} = A \cos x + B \sin x$	2	SR If some or all of the working is omitted, award full credit for correct answer
(ii)(a)	$y = p(\ln \sin x) \sin x + qx \cos x$	M1	For attempting to differentiate P.I. (product rule needed at least once)
	$\frac{dy}{dx} = p \frac{\cos x}{\sin x} \sin x + p(\ln \sin x) \cos x + q \cos x - qx \sin x$	A1	For correct (unsimplified) result AEF
	$\frac{d^2 y}{dx^2} = -p \sin x - p(\ln \sin x) \sin x + \frac{p \cos^2 x}{\sin x} - 2q \sin x - qx \cos x$	A1	For correct (unsimplified) result AEF
	$-p \sin x + \frac{p \cos^2 x}{\sin x} - 2q \sin x \equiv \frac{1}{\sin x}$	M1	For substituting their $\frac{d^2 y}{dx^2}$ and $y$ into D.E.
		M1	For using $\sin^2 x + \cos^2 x = 1$
	$\Rightarrow p - 2(p + q) \sin^2 x \equiv 1$	A1	6
(b)		M1	For attempting to find $p$ and $q$ by equating coefficients of constant and $\sin^2 x$ AND/OR giving value(s) to $x$ (allow any value for $x$ , including 0)
	$p = 1, q = -1$	A1	2
(iii)	G.S. $y = A \cos x + B \sin x + (\ln \sin x) \sin x - x \cos x$	B1√	For correct G.S. f.t. from their C.F. and P.I. with 2 arbitrary constants in C.F. (allow given form of P.I. if $p$ and $q$ have not been found)
	cosec $x$ undefined at $x = 0, \pi, 2\pi$	M1	For considering domain of cosec $x$ OR $\sin x \neq 0$ OR $\ln \sin x$ term
	OR $\sin x > 0$ in $\ln \sin x$		
	$\Rightarrow 0 < x < \pi$	A1	3
			For stating correct range CAO SR Award B1 for correct answer with justification omitted or incorrect

# 4728 Mechanics 1

<b>1(i)</b>	900a = 600 - 240 a = 0.4 ms <sup>-2</sup> AG	M1 A1 [2]	N2L with difference of 2 forces, accept 360
<b>(ii)</b>	9 = 5 + 0.4t t = 10 s 9 <sup>2</sup> = 5 <sup>2</sup> + 2x0.4s s = 70 m	M1 A1 M1 A1 [4]	v = u + 0.4t or v = u + (cv 0.4)t or s=(u+v)t/2 or s=ut+0.5xcv(0.4)t <sup>2</sup>
<b>2(i)</b>	Resolves a force in 2 perp. directions Uses Pythagoras R <sup>2</sup> = (14sin30) <sup>2</sup> + (12+14cos30) <sup>2</sup> {or R <sup>2</sup> = (12sin30) <sup>2</sup> + (14+12cos30) <sup>2</sup> } R = 25.1 AG	M1* D*M1 A1 A1	Uses vector addition or subtraction Uses cosine rule R <sup>2</sup> = 14 <sup>2</sup> + 12 <sup>2</sup> - 2x14x12cos150
<b>(ii)</b>	Trig to find angle in a valid triangle tanB=7/24.1, sinB=7/25.1, cosB=24.1/25.1 B = 016, (0)16.1° or (0)16.2°	A1 [5] M1 A1 A1 [3]	cso (Treat R <sup>2</sup> = 14 <sup>2</sup> + 12 <sup>2</sup> + 2x14x12cos30 as correct) Angle should be relevant sinB/14 = sin150/25.1. Others possible. Cosine rule may give (0)16.4, award A1
<b>3(i)</b>	a = 6/5 a = 1.2 ms <sup>-2</sup>	M1 A1	Acceleration is gradient idea, for portion of graph Accept 6/5
<b>(ii)</b>	s = (6x10/2) x2 s = 60 m	[2] M1 M1 A1 [3]	Area under graph idea or a formula used correctly Double {Quadruple} journey
<b>(iii)</b>	v = -6 + 1.2(17-15) v = -3.6 ms <sup>-1</sup>	M1 A1 A1 [3]	v=u+at idea, t not equal to 17 (except v=1.2t-24) 0 = v + cv(1.2)(20-17), v <sup>2</sup> -2.4v -21.6 = 0, etc <b>SR</b> v=3.6 neither A1, but give both A1 if final answer given is -3.6
<b>4(i)</b>	F = 15sin50 - 15sin30 = 3.99 N Left	M1 A1	Difference of 2 horizontal components, both < 15 Not 4 or 4.0
<b>(ii)</b>	R = f(30, 15cos50, 15cos30) R = 30-15cos50-15cos30 μ = 3.99/7.36(78 ) μ = 0.541 or 0.542 or 0.543	B1 [3] M1 A1 A1 M1 A1 [5]	Accept reference to 30 degree string May be given in ii if not attempted in i Equating 4 vertical forces/components 30g is acceptable =7.36(78..), treat 30g as a misread Using F = μR, with cv(3.99) and cv(7.36(78..)) Accept 0.54 from correct work, e.g. 4/7.4
<b>5(i)</b>	2400x5 - 3600x3 2400v + 3600v 2400x5 - 3600x3 = 2400v + 3600v v = 0.2 ms <sup>-1</sup> B	B1 B1 M1 A1 B1 [5]	Award if g included Award if g included Equating momentums (award if g included) Not given if g included or if negative.
<b>(ii)(a)</b>	+/-(-2400v + 3600v) 2400x5 - 3600x3 = -2400v + 3600v v = 1 ms <sup>-1</sup>	B1 M1 A1	No marks in (ii) if g included Equating momentums if "after" signs differ Do not accept if - sign "lost"
<b>(b)</b>	I = 2400 x (5+/-1) or 3600 x (3+/-1) I = 14400 kgms <sup>-1</sup>	M1 A1 [5]	Product of either mass and velocity change Accept -14400

<p><b>6(i)</b></p> <p><math>x = 0.01t^4 - 0.16t^3 + 0.72t^2</math>  <math>v = dx/dt</math>  <math>v = 0.04t^3 - 0.48t^2 + 1.44t</math>  <math>v(2) = 1.28 \text{ ms}^{-1}</math></p>	<p>AG</p>	<p>M1 A1 A1 [3]</p>	<p>Uses differentiation, ignore +c          or <math>v = 4(0.01t^3) - 3(0.16t^2) + 2(0.72t)</math>          Evidence of evaluation needed</p>
<p><b>(ii)</b></p> <p><math>a = dv/dt</math>  <math>a = 0.12t^2 - 0.96t + 1.44</math>  <math>t^2 - 8t + 12 = 0</math></p>	<p>AG</p>	<p>M1 A1 A1 [3]</p>	<p>Uses differentiation          or <math>a = 3(0.04t^2) - 2(0.48t) + 1.44</math>          Simplifies <math>0.12t^2 - 0.96t + 1.44 = 0</math>, (or verifies the roots of QE make acceleration zero)</p>
<p><b>(iii)</b></p> <p><math>(t - 2)(t - 6) = 0</math>  <math>t = 2</math>  <math>t = 6</math>  <math>v(6) = 0 \text{ ms}^{-1}</math></p>		<p>M1 A1 A1 B1 [4]</p>	<p>Solves quadratic (may be done in ii <u>if used to find v(6)</u>)          Or <i>Factorises v into 3 linear factors</i> M1  <math>v = 0.04t(t-6)^2</math> A1 <i>Identifies t=6</i> A1          Evidence of evaluation needed</p>
<p><b>(iv)</b></p> <p>Away from A</p>		<p>B1 B1 B1 B1 [4]</p>	<p>Starts at origin          Rises to single max, continues through single min          Minimum on t axis, non-linear graph</p>
<p><b>(v)</b></p> <p><math>AB = 0.01x6^4 - 0.16x6^3 + 0.72x6^2</math>  <math>AB = 4.32 \text{ m}</math></p>		<p>M1 A1 [2]</p>	<p>Or integration of v(t), with limits 0, 6 or substitution, using cv(6) from iii</p>

<p><b>7(i)</b></p> <p><math>(R=)0.2x9.8\cos45</math>  <math>F=1xR=1x.2x9.8\cos45=1.386 \text{ N}</math></p>	<p>AG</p>	<p>M1 A1 [2]</p>	<p>Not <math>F = 0.2x9.8\cos45</math> or <math>0.2x9.8\sin 45</math> unless followed by (eg) <math>Fr = 1x F = 1.386</math> when M1A1</p>
<p><b>(ii)</b></p> <p>Any 1 application of N2L // to plane with correct mass and number of forces  <math>0.4a = 0.2g\sin45 + 0.2g\sin45 - 1.38(592..)</math>  <math>a = 3.465 \text{ ms}^{-2}</math> AG  <math>0.2a = 0.2g\sin45 - T</math> or  <math>0.2a = T + [0.2g\sin45 - 1.38(592..)]</math>  <math>T = 0.693 \text{ N}</math></p> <p>OR</p> <p>Any 1 application of N2L // to plane with correct mass and number of forces  <math>0.2a = 0.2g\sin45 - T</math> or  <math>0.2a = T + [0.2g\sin45 - 1.38(592..)]</math>          Eliminates a or T  <math>a = 3.465 \text{ ms}^{-2}</math> AG  <math>T = 0.693 \text{ N}</math></p>		<p>M1 A1 A1 M1 A1 [5]</p>	<p>Must use component of weight</p> <p>Accept with 3.465 (or close) instead of a          Accept omission of [term] for M1          Accept 0.69</p> <p>Must use component of weight          Either correct          Both correct. Accept omission of [term] for A1 only</p>
<p><b>(iii)</b></p> <p><math>v^2 = 2 \times 3.465 \times 0.5</math>  <math>v = 1.86 \text{ ms}^{-1}</math></p>		<p>M1 A1 [2]</p>	<p>Using <math>v^2 = 0^2 + 2xv(3.465)s</math></p>
<p><b>(iv)</b></p> <p>For Q  <math>(0.2)a = (0.2)g\sin45 - (1)(0.2)g\cos45</math>  <math>a=0</math> [AG]  <math>T = (3/1.86) = 1.6(12)</math></p> <p>For P  <math>a = 9.8\sin45</math>  <math>2.5 = 1.86(14..t) + 0.5 \times (9.8\sin45)t^2</math>  <math>t = 0.6(223)</math>          time difference <math>1.612 - 0.622 = 0.99(0) \text{ s}</math></p>		<p>M1 A1 B1 B1 M1 A1 A1 [7]</p>	<p>Attempting equation to find a for Q          Accept from <math>0.2g\sin45 - 1.386</math>          Accept 2 sf</p> <p><math>a = 6.93</math>          Using <math>2.5 = cv(1.86)t + 0.5cv(6.93)t^2</math> [not 9.8 or 3.465]          Accept 1sf          Accept art 0.99 from correct work</p>

## 4729 Mechanics 2

<b>1</b>	$200\cos 35^\circ$ $200\cos 35^\circ \times d = 5000$ $d = 30.5 \text{ m}$	B1 M1 A1 <b>3</b>		<b>3</b>
<b>2</b>	$0.03R = \frac{1}{2} \times 0.009(250^2 - 150^2)$ $0.03R$	M1 B1	$150^2 = 250^2 + 2a \times 0.03$ $a = \pm 2 \times 10^6 / 3$ or $\pm 666,667$ (A1)	
	either K.E. $R = 6000 \text{ N}$	B1 A1 <b>4</b>	$F = 0.009a$ (M1) <b>4</b> unit errors	<b>4</b>
<b>3 (i)</b>	$D = 12000/20$ $12000/20 = k \times 20 + 600 \times 9.8 \times 0.1$ $k = 0.6$	B1 M1 A1 <b>3</b>	<b>AG</b>	
<b>(ii)</b>	$16000/v = 0.6v + 600 \times 9.8 \times 0.1$ $0.6v^2 + 588v - 16000 = 0$ $v = 26.5 \text{ m s}^{-1}$	M1 M1 A1 <b>3</b>	attempt to solve quad. (3 terms)	
<b>(iii)</b>	$16000/32 - 0.6 \times 32 = 600a$ $a = 0.801 \text{ m s}^{-2}$	M1 A1 A1 <b>3</b>	0.80 or 0.8	<b>9</b>
<b>4 (i)</b>	$0 = 35\sin\theta \times t - 4.9t^2$ $t = 35\sin\theta/4.9$ $50\sin\theta/7$ $R = 35\cos\theta \times t$ $aef$  $R = 35^2 \sin\theta \cdot \cos\theta / 4.9$  $R = 125\sin 2\theta$	M1 A1 B1  M1  A1 <b>5</b>	$R = u^2 \sin 2\theta / g$ only ok if proved or $70\sin\theta / g$ aef  their t  eliminate t  <b>AG</b>	
<b>(ii)</b>	$110 = 125\sin 2\theta$ $\theta = 30.8^\circ$ or $59.2^\circ$ $t = 3.66 \text{ s}$ or $6.13 \text{ s}$	M1 A1+1 A1+1 <b>5</b>		<b>10</b>
<b>5 (i)</b>	$3/8 \times 3$ (1.125) $0.53d = 5 \times 0.02 + (10 + 3/8 \times 3) \times 0.5$	B1 M1 A1 A1 <b>4</b>	c.o.m. hemisphere $0.53e = 3 \times 5/8 \times 0.5 + 8 \times 0.02 + 13 \times .01$ $0.53f = 3 \times 3/8 \times 0.5 - 5 \times 0.02 - 10 \times 0.01$ <b>AG</b> ( $e = 2.316$ $f = 0.684$ ) distance / angle not a complimentary pair	
<b>(ii)</b>	Attempt to calc a pair relevant to P,G $OP = 0.9$ (pair), $p = 73.3^\circ$ $q = 16.7^\circ$ $r = 76.9^\circ$ $(77.2^\circ)$ , $s = 13.1^\circ$ ( $12.8^\circ$ ) $AC = 0.86$ , $BC = 0.67$ , $AD = 10.4$ $BD = 10.2$ $r > p$ , $s < q$ , $p + s < 90$ , $0.67 < 0.86$ , $10.2 < 10.4$ it is in equilibrium	M1 A1 <b>4</b>	make relevant comparison $0.7 < 0.9$ ( $OG < OP$ ) $10.7 < 10.9$	<b>8</b>

<p><b>6 (i)</b></p> <p><math>T\cos 60^\circ = S\cos 60^\circ + 4.9</math></p> <p><math>T\sin 60^\circ + S\sin 60^\circ = 0.5 \times 3^2/0.4</math></p> <p><math>(S + 9.8)\sin 60^\circ + S\sin 60^\circ = 45/4</math></p> <p><math>S = 1.60 \text{ N}</math></p> <p><math>T = 11.4 \text{ N}</math></p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1 7</p>	<p>Resolving vertically nb for M1: (must be components – all 4 cases) Res. Horiz. <math>m\omega^2</math> ok if <math>\omega \neq 3</math> If equal tensions <math>2T=45/4</math> M1 only</p>	12
<p><b>(ii)</b></p> <p><math>T\cos 60^\circ = 4.9</math></p> <p><math>T = 9.8</math></p> <p><math>T\sin 60^\circ = 0.5 \times 0.4\omega^2</math></p> <p><math>\omega = 6.51 \text{ rad s}^{-1}</math></p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1 5</p>	<p>Resolving vertically (component)</p> <p>Resolving horiz. (component)</p> <p>or 6.5</p>	
<p><b>7 (i)</b></p> <p><math>u = 3 \text{ m s}^{-1}</math></p> <p><math>6 = 2x + 3y</math></p> <p><math>e = (y - x) / 3</math></p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1 6</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 7</p>	<p>(<math>e = 2/3</math>) (equus must be consistent)</p> <p><b>AG</b></p> <p>or (B1) <math>\frac{1}{2}mx^2</math></p> <p>(B1) <math>\frac{1}{2}mv^2</math></p> <p>(B1) <math>mx9.8x4</math></p> <p><math>v = \sqrt{(2^2 + 2x9.8x4)}</math></p> <p>or <math>\cos^{-1}(2/9.08)</math></p> <p>12.7° to vertical</p>	13
<p><b>8 (i)</b></p> <p>com of <math>\Delta</math> 3 cm right of C</p> <p><math>(48+27)\bar{x} = 48x4 + 27x11</math></p> <p><math>\bar{x} = 6.52</math></p> <p>com of <math>\Delta</math> 2 cm above AD</p> <p><math>(48+27)\bar{y} = 48x3 + 27x2</math></p> <p><math>\bar{y} = 2.64</math></p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1 8</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1 5</p>	<p>can be implied e.g. <math>7/\sin 30^\circ \cdot F</math></p> <p>7.034 (AG) or <math>(6.52 - 2.64\tan 30^\circ)</math></p> <p>52.0° (GAH) or (above)<math>x\cos 30^\circ</math> <math>(5.00)x\cos 30^\circ</math> (4.33)</p> <p><math>14F = 3x9.8x7.034x\cos 52.0^\circ</math></p>	13
<p><b>(ii)</b></p> <p>14F</p> <p><math>3g\cos 30^\circ \times 6.52</math></p> <p><math>3g\sin 30^\circ \times 2.64</math></p> <p><math>14F = 3g\cos 30^\circ \times 6.52 - 3g\sin 30^\circ \times 2.64</math></p> <p><math>F = 9.09 \text{ N}</math></p>			

## 4730 Mechanics 3

1	(i) $T = (1.35mg)(3 - 1.8) \div 1.8$ [ $0.9mg = ma$ ] Acceleration is $8.82ms^{-2}$	B1 M1 A1	3	For using $T = ma$
	(ii) Initial EE = $(1.35mg)(3 - 1.8)^2 \div (2 \times 1.8)$ [ $\frac{1}{2}mv^2 = 0.54mg$ ] Speed is $3.25ms^{-1}$	B1 M1 A1	3	For using $\frac{1}{2}mv^2 = \text{Initial EE}$
2	(i) Component is $8\sin 27^\circ$ Component is $2.18ms^{-1}$	M1 A1 A1	3	For using NEL vertically
	(ii) Change in velocity vertically = $8\sin 27^\circ(1 + e)$	B1ft		ft $8\sin 27^\circ + \text{candidate's ans. in (i)}$ For using $ I  = m \times \text{change in velocity}$
	$ I  = 0.2 \times 5.81$ Magnitude of Impulse is $1.16 \text{ kgms}^{-1}$	M1 A1ft	3	ft incorrect ans. in (i) providing both M marks are scored.
3	$0.8 \times 12 \cos 60^\circ = 0.8a + 2b$	M1 A1		For using the principle of conservation of momentum in the <b>i</b> direction
	$0.75 \times 12 \cos 60^\circ = b - a$	M1 A1		For using NEL
	[ $4.8 = 0.8a + 2(a + 4.5)$ ] $a = -1.5$	DM1 A1		For eliminating b; depends on at least one previous M mark
	Comp. of vel. perp. to l.o.c. after impact is $12\sin 60^\circ$	B1		
	The speed of A is $10.5ms^{-1}$	M1 A1ft		For correct method for speed or direction ft $v^2 = a^2 + 108$
	Direction of A is at $98.2^\circ$ to l.o.c.	A1ft	10	Accept $\theta = 81.8^\circ$ if $\theta$ is clearly and appropriately indicated; ft $\tan^{-1} \theta = (12\sin 60^\circ)/ a $

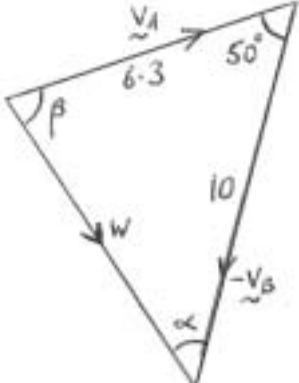

4	(i)	$[mgsin \alpha - 0.2mv = ma]$	M1	For using Newton's second law	
		$5 \frac{dv}{dt} = 28 - v$	A1	AG	
		$[\int \frac{5}{28 - v} dv = \int dt]$	M1	For separating variables and integrating	
		$(C) - 5\ln(28 - v) = t$	A1		
			M1	For using $v = 0$ when $t = 0$	
			A1ft	ft for $\ln[(28 - v)/28] = t/A$ from $C + A\ln(28 - v) = t$ previously	
		$\ln[(28 - v)/28] = -t/5$	M1	For expressing $v$ in terms of $t$	
		$[28 - v = 28e^{-t/5}]$	M1	ft for $v = 28(1 - e^{-t/5})$ from	
		$v = 28(1 - e^{-t/5})$	A1ft	8	$\ln[(28 - v)/28] = t/A$ previously
	(ii)				For using $a = (28 - v(t))/5$ or $a = d(28 - 28e^{-t/5})/dt$ and substituting $t = 10$ .
		$[a = 28e^{-t/5}/5]$	M1		ft from incorrect $v$ in the form $a + be^{ct}$ ( $b \neq 0$ ); Accept $5.6/e^2$
		Acceleration is $0.758ms^{-2}$	A1ft	2	
5	(i)		M1	For taking moments about B or about A for the whole or For taking moments about X for the whole and using $R_A + R_B = 280$ and $F_A = F_B$	
		$1.4R_A = 150 \times 0.95 + 130 \times 0.25$ or			
		$1.4R_B = 130 \times 1.15 + 150 \times 0.45$ or			
		$1.2F - 0.9(280 - R_B) + 0.45 \times 150 - 1.2F +$	A1		
		$0.5R_B$			
		$-0.25 \times 130 = 0$	A1	AG	
		$R_A = 125N$	A1		
		$R_B = 155N$	B1	4	
	(ii)		M1	For taking moments about X for XA or XB	
		$1.2F_A = -150 \times 0.45 + 0.9R_A$ or	A1		
		$1.2F_B = 0.5R_B - 130 \times 0.25$	A1ft	$F_B = (1.25R_B - 81.25)/3$	
	$F_A$ or $F_B = 37.5N$	A1ft			
	$F_B$ or $F_A = 37.5N$	B1ft	4		
(iii)	Horizontal component is $37.5N$ to the left	B1ft		ft $H = F$ or $H = 56.25 - 0.75V$ or $12H = 325 + 5V$	
	$[Y + R_A = 150]$	M1	For resolving forces on XA vertically		
	Vertical component is $25N$ upwards	A1ft	3	ft $3V = 225 - 4H$ or $V = 2.4H - 65$	

6	(i)			For applying Newton's second law
		$[0.36 - 0.144x = 0.1a]$	M1	
		$\ddot{x} = 3.6 - 1.44x$	A1	
		$\ddot{y} = -1.44y \rightarrow \text{SHM}$	or	
		$d^2(x - 2.5) / dt^2 = -1.44(x - 2.5) \rightarrow \text{SHM}$	B1	
			M1	For using $T = 2\pi / n$
		Of period 5.24s	A1	5 AG
	(ii)	Amplitude is 0.5m	B1	
		$0.48^2 = 1.2^2(0.5^2 - y^2)$	M1	For using $v^2 = n^2(a^2 - y^2)$
		Possible values are 2.2 and 2.8	A1ft	
		$[t_0 = (\sin^{-1}0.6)/1.2; t_1 = (\cos^{-1}0.6)/1.2]$	A1	4
	(iii)		M1	For using $y = 0.5\sin 1.2t$ to find $t_0$ or $y = 0.5\cos 1.2t$ to find $t_1$
		$t_0 = 0.53625 \dots$ or $t_1 = 0.7727 \dots$	A1	Principal value may be implied
	(a)	$[2(\sin^{-1}0.6)/1.2$ or $(\pi - 2\cos^{-1}0.6)/1.2]$	M1	For using $\Delta t = 2t_0$ or $\Delta t = T/2 - 2t_1$
		Time interval is 1.07s	A1ft	ft incorrect $t_0$ or $t_1$
	(b)		M1	From $\Delta t = T/2 - 2t_0$ or $\Delta t = 2t_1$ ; ft 2.62 - ans(a) or incorrect $t_0$ or $t_1$
		Time interval is 1.55s	B1ft	5
7	(i)		M1	For using KE gain = PE loss
		$\frac{1}{2}mv^2 = mga(1 - \cos\theta)$	A1	
		$aw^2 = 2g(1 - \cos\theta)$	B1	3 AG From $v = wr$
	(ii)			For using Newton's second law radially (3 terms required) with accel = $v^2/r$ or $w^2r$
		$mv^2/a = mg\cos\theta - R$ or $maw^2 = mg\cos\theta - R$	M1	
		$[2mg(1 - \cos\theta) = mg\cos\theta - R]$	A1	
		$R = mg(3\cos\theta - 2)$	DM1	For eliminating $v^2$ or $w^2$ ; depends on at least one previous M1
			A1ft	4 ft sign error in N2 equation
	(iii)			For using Newton's second law tangentially or differentiating
		$[mg\sin\theta = m(\text{accel.})$ or $2a(\dot{\theta})\ddot{\theta} = 2g\sin\theta(\dot{\theta})]$	M1	$aw^2 = 2g(1 - \cos\theta)$ w.r.t. t
		Accel. ( $=a\ddot{\theta}$ ) = $g\sin\theta$	A1	
		$[\theta = \cos^{-1}(2/3)]$	M1	For using $R = 0$
		Acceleration is $7.30\text{ms}^{-2}$	A1ft	4 ft from incorrect R of the form $mg(A\cos\theta + B)$ , $A \neq 0$ , $B \neq 0$ ; accept $g\sqrt{5}/3$
	(iv)			For using rate of change = $(dR/d\theta)(d\theta/dt)$
		$dR/dt = (-3mg\sin\theta)\sqrt{2g(1 - \cos\theta)}/a$	M1	
			A1ft	ft from incorrect R of the form $mg(A\cos\theta + B)$ , $A \neq 0$
		Rate of change is $-mg\sqrt{\frac{10g}{3a}}\text{Ns}^{-1}$	M1	For using $\cos\theta = 2/3$
			A1ft	4 Any correct form of $\dot{R}$ with $\cos\theta = 2/3$ used; ft with $\square$ from incorrect R of the form $mg(A\cos\theta + B)$ , $A \neq 0$ , $B \neq 0$



## 4731 Mechanics 4

1	By conservation of angular momentum $1.5 \times 21 + I_G \times 36 = 1.5 \times 28 + I_G \times 34$ $I_G = 5.25 \text{ kg m}^2$	M1 A1A1 A1 <b>4</b>	Give A1 for each side of the equation or $1.5(28 - 21) = I_G(36 - 34)$
2 (i)	Using $\omega_1^2 = \omega_0^2 + 2\alpha\theta$ , $0^2 = 8^2 + 2\alpha(2\pi \times 16)$ $\alpha = -\frac{1}{\pi} = -0.318$ Angular deceleration is $0.318 \text{ rad s}^{-2}$	M1 A1 <b>2</b>	Accept $-\frac{1}{\pi}$
(ii)	Using $\omega_1^2 = \omega_0^2 + 2\alpha\theta$ , $\omega^2 = 8^2 + 2\alpha(2\pi \times 15)$ $\omega = 2 \text{ rad s}^{-1}$	M1 A1 ft <b>2</b>	or $0^2 = \omega^2 + 2\alpha(2\pi)$ ft is $\sqrt{64 - 60\pi \alpha }$ or $\sqrt{4\pi \alpha }$ Allow A1 for $\omega = 2$ obtained using $\theta = 16$ and $\theta = 15$ (or $\theta = 1$ )
(iii)	Using $\omega_1 = \omega_0 + \alpha t$ , $0 = \omega + \alpha t$ $t = 2\pi = 6.28 \text{ s}$	M1 A1 ft <b>2</b>	or $2\pi = 0t - \frac{1}{2}\alpha t^2$ ft is $\frac{\omega}{ \alpha }$ or $\sqrt{\frac{4\pi}{ \alpha }}$ Accept $2\pi$
3	$A = \int_0^3 (2x + x^2) dx$ $= \left[ x^2 + \frac{1}{3}x^3 \right]_0^3 = 18$ $A\bar{x} = \int_0^3 x(2x + x^2) dx$ $= \left[ \frac{2}{3}x^3 + \frac{1}{4}x^4 \right]_0^3 = \frac{153}{4} = 38.25$ $\bar{x} = \frac{38.25}{18} = \frac{17}{8} = 2.125$ $A\bar{y} = \int_0^3 \frac{1}{2}(2x + x^2)^2 dx$ $= \int_0^3 (2x^2 + 2x^3 + \frac{1}{2}x^4) dx$ $= \left[ \frac{2}{3}x^3 + \frac{1}{2}x^4 + \frac{1}{10}x^5 \right]_0^3 = 82.8$ $\bar{y} = \frac{82.8}{18} = 4.6$	M1 A1 M1 M1 A1 M1 M1 M1 A1 <b>9</b>	Definite integrals may be evaluated by calculator (i.e with no working shown) Integrating and evaluating (dependent on previous M1) or $\int_0^{15} (3 - (\sqrt{y+1} - 1)) y dy$ Arranging in integrable form Integrating and evaluating SR If $\frac{1}{2}$ is missing, then M0M1M1A0 can be earned for $\bar{y}$

<p>4 (i)</p>	 <p> <math>w^2 = 6.3^2 + 10^2 - 2 \times 6.3 \times 10 \cos 50^\circ</math>  <math>w = 7.66 \text{ ms}^{-1}</math>  <math>\frac{\sin \alpha}{6.3} = \frac{\sin 50^\circ}{w}</math>  <math>\alpha = 39.04^\circ \quad (\beta = 90.96^\circ)</math>                      Bearing is <math>205 - \alpha = 166^\circ</math> </p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>5</p>	<p>Correct velocity triangle</p> <p><i>This mark cannot be earned from work done in part (ii)</i></p>
	<p>OR</p> <p> <math>\begin{pmatrix} 6.3 \sin 75 \\ 6.3 \cos 75 \end{pmatrix} - \begin{pmatrix} 10 \sin 25 \\ 10 \cos 25 \end{pmatrix} = \begin{pmatrix} 1.859 \\ -7.433 \end{pmatrix}</math> </p> <p> <math>w = \sqrt{1.859^2 + 7.433^2} = 7.66</math>                      Bearing is <math>180 - \tan^{-1} \frac{1.859}{7.433} = 166^\circ</math> </p>	<p>M1A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>Finding magnitude or direction</p>
<p>(ii)</p>	<p>As viewed from B</p>  <p> <math>d = 2500 \sin 14.04</math>  <math>= 607 \text{ m}</math> </p>	<p>B1 ft</p> <p>M1</p> <p>A1</p> <p>3</p>	<p>Diagram showing path of A as viewed from B <i>May be implied</i>                      Or B1 for a correct (ft) expression for <math>d^2</math> in terms of <math>t</math></p> <p>or other complete method                      Accept 604.8 to 609                      SR If <math>\beta = 89^\circ</math> is used, give A1 for 684.9 to 689.1</p>

<b>5 (i)</b>	$V = \int_a^{4a} \pi(ax) dx$ $= \left[ \frac{1}{2} \pi a x^2 \right]_a^{4a} = \frac{15}{2} \pi a^3$ <p>Hence <math>m = \frac{15}{2} \pi a^3 \rho</math></p> $I = \sum \frac{1}{2} (\rho \pi y^2 \delta x) y^2 = \int \frac{1}{2} \rho \pi y^4 dx$ $= \int_a^{4a} \frac{1}{2} \rho \pi a^2 x^2 dx$ $= \left[ \frac{1}{6} \rho \pi a^2 x^3 \right]_a^{4a} = \frac{21}{2} \rho \pi a^5$ $= \frac{7}{5} \left( \frac{15}{2} \pi a^3 \rho \right) a^2 = \frac{7}{5} m a^2$	M1 M1 M1 M1 A1 A1 ft A1 A1 (ag)	<i>(Omission of <math>\pi</math> is an accuracy error)</i>  For $\int y^4 dx$  Substitute for $y^4$ and correct limits	<b>8</b>
<b>(ii)</b>	MI about axis, $I_A = \frac{7}{5} m a^2 + m a^2$ $= \frac{12}{5} m a^2$ <p>Period is <math>2\pi \sqrt{\frac{I}{mgh}}</math></p> $= 2\pi \sqrt{\frac{\frac{12}{5} m a^2}{mga}} = 2\pi \sqrt{\frac{12a}{5g}}$	M1 A1 M1 A1 ft	Using parallel axes rule  ft from any $I$ with $h = a$	<b>4</b>
<b>6 (i)</b>	$I = \frac{1}{3} m \left\{ a^2 + \left( \frac{3}{2} a \right)^2 \right\} + m \left( \frac{1}{2} a \right)^2$ $= \frac{13}{12} m a^2 + \frac{1}{4} m a^2 = \frac{4}{3} m a^2$	M1 M1 A1 (ag)	MI about perp axis through centre Using parallel axes rule	<b>3</b>
<b>(ii)</b>	By conservation of energy $\frac{1}{2} \left( \frac{4}{3} m a^2 \right) \omega^2 - \frac{1}{2} \left( \frac{4}{3} m a^2 \right) \frac{9g}{10a} = mg \left( \frac{1}{2} a - \frac{1}{2} a \times \frac{3}{5} \right)$ $\frac{2}{3} m a^2 \omega^2 - \frac{3}{5} m g a = \frac{1}{5} m g a$ $\omega^2 = \frac{6g}{5a}$	M1 A1 A1 (ag)	Equation involving KE and PE	<b>3</b>
<b>(iii)</b>	$mg \cos \theta - R = m \left( \frac{1}{2} a \right) \omega^2$ $mg \times \frac{3}{5} - R = \frac{3}{5} mg$ $R = 0$ $mg \left( \frac{1}{2} a \sin \theta \right) = I \alpha$ $\alpha = \frac{3g}{10a}$ $mg \sin \theta - S = m \left( \frac{1}{2} a \right) \alpha$ $S = \frac{4}{5} mg - \frac{3}{20} mg$ $= \frac{13}{20} mg$	M1 A1 A1 (ag) M1A1 A1 M1A1 A1	Acceleration $r\omega^2$ and three terms (one term must be $R$ ) SR $mg \cos \theta + R = m \left( \frac{1}{2} a \right) \omega^2 \Rightarrow R = 0$ earns M1A0A1 Applying $L = I\alpha$  Acceleration $r\alpha$ and three terms (one term must be $S$ ) or $S \left( \frac{1}{2} a \right) = I_G \alpha = \frac{13}{12} m a^2 \alpha$	<b>9</b>

7 (i)	$U = 3mgx + 2mg(3a - x)$ $+ \frac{mg}{2a}(x - a)^2 + \frac{2mg}{2a}(2a - x)^2$ $= \frac{mg}{2a}(3x^2 - 8ax + 21a^2)$ $\frac{dU}{dx} = 3mg - 2mg + \frac{mg}{a}(x - a) - \frac{2mg}{a}(2a - x)$ $= \frac{3mgx}{a} - 4mg$ <p>When <math>x = \frac{4}{3}a</math>, <math>\frac{dU}{dx} = 4mg - 4mg = 0</math> so this is a position of equilibrium</p> $\frac{d^2U}{dx^2} = \frac{3mg}{a}$ $> 0$ , so equilibrium is stable	B1B1 B1B1 M1 A1 A1 (ag) M1 A1 (ag)	<p>Can be awarded for terms listed separately</p> <p>Obtaining <math>\frac{dU}{dx}</math> (or any multiple of this)</p> <p>9</p>
(ii)	<p>KE is <math>\frac{1}{2}(3m)v^2 + \frac{1}{2}(2m)v^2</math></p> <p>Energy equation is <math>U + \frac{5}{2}mv^2 = \text{constant}</math></p> <p>Differentiating with respect to <math>t</math></p> $\left(\frac{3mgx}{a} - 4mg\right) \frac{dx}{dt} + 5mv \frac{dv}{dt} = 0$ $\frac{3gx}{a} - 4g + 5 \frac{d^2x}{dt^2} = 0$ <p>Putting <math>x = \frac{4}{3}a + y</math>, <math>\frac{3gy}{a} + 5 \frac{d^2y}{dt^2} = 0</math></p> $\frac{d^2y}{dt^2} = -\frac{3g}{5a}y$ <p>Hence motion is SHM with period <math>2\pi \sqrt{\frac{5a}{3g}}</math></p>	M1A1 M1 A1 ft A1 ft M1A1 ft A1 (ag) A1	<p>Differentiating the energy equation (with respect to <math>t</math> or <math>x</math>)</p> <p>Condone <math>\ddot{x}</math> instead of <math>\ddot{y}</math> Award M1 even if KE is missing</p> <p>Must have <math>\ddot{y} = -\omega^2 y</math> or other satisfactory explanation</p> <p>9</p>

# 4732 Probability & Statistics 1

Note: “(3 sfs)” means “answer which rounds to ... to 3 sfs”. If correct ans seen to  $\geq 3$ sfs, ISW for later rounding  
 Penalise over-rounding only once in paper.

<b>1(i)</b>	(a) -1 (b) 0	B1 B1 2	allow $\approx -1$ or close to -1 not “strong corr’n”, not -0.99 allow $\approx 0$ or close to 0 not “no corr’n”
<b>(ii)</b>	$\begin{array}{cccc} 4 & 3 & 2 & 1 \\ 1 & 3 & 4 & 2 \end{array}$ or $\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{array}$ $\Sigma d^2 = 14$ $1 - \frac{6\Sigma d^2}{4(4^2-1)}$ $= -0.4 \text{ oe}$	M1 A1 M1 M1 A1 5	Ranks attempted, even if opp  Dep M1 or $S_{xy} = 23^{-100/4}$ or $S_{xx} = S_{yy} = 30^{-100/4}$ Dep 2 <sup>nd</sup> M1 $S_{xy}/\sqrt{(S_{xx}S_{yy})}$
<b>Total</b>		<b>7</b>	
<b>2(i)</b>	$\frac{{}^7C_2 \times {}^8C_3}{{}^{15}C_5}$ $= \frac{56}{143} \text{ or } \frac{1176}{3003} \text{ or } 0.392 \text{ (3sfs)}$	M1 M1  A1 3	${}^7C_2 \times {}^8C_3$ or 1176 : M1 $(\text{Any C or P})/{}^{15}C_5$ : M1 (dep < 1)  or $\frac{7}{15} \times \frac{6}{14} \times \frac{8}{13} \times \frac{7}{12} \times \frac{6}{11}$ or 0.0392: M1 $\times {}^5C_2$ or $\times 10$ : M1 (dep $\geq 4$ probs mult) if 2 $\leftrightarrow$ 3, treat as MR max M1M1
<b>(ii)</b>	$3! \times 2!$ or ${}^3P_3 \times {}^2P_2$ not in denom $= 12$	M1 A1 2	BABAB seen: M1  120-12: M1A0 NB $4!/2! = 12$ : M0A0
<b>Total</b>		<b>5</b>	
<b>3(i)(a)</b>	0.9368 or 0.937	B1 1	
<b>(b)</b>	$0.7799 - 0.5230$ or ${}^8C_5 \times 0.45^3 \times 0.55^5$ $= 0.2569$ or 0.2568 or 0.257	M1 A1 2	Allow 0.9368 – 0.7799
<b>(c)</b>	0.7799 seen – 0.0885 (not 1 – 0.0885) $= 0.691$ (3 sfs)	M1 M1 A1 3	${}^8C_5 \times 0.45^3 \times 0.55^5 + {}^8C_4 \times 0.45^4 \times 0.55^4 + {}^8C_3 \times 0.45^5 \times 0.55^3$ : M2 1 term omitted or wrong or extra: M1
<b>(ii)(a)</b>	${}^{10}C_2 \times (7/12)^8 \times (5/12)^2$ seen $= 0.105$ (3 sfs)	M1 A1 2	or 0.105 seen, but not ISW for A1
<b>(b)</b>	$2^{31/72}$ or $175/72$ or 2.43 (3 sfs)	B1 1	NB $12/5 = 2.4$ : B0
<b>Total</b>		<b>9</b>	
<b>4(i)</b>	$1/20 \times 1/10$ or $1/200$ or 0.005 $\times 2$ $= 1/100$ or 0.01	M1 M1dep A1 3	
<b>(ii)</b>	$E(X) = 0 + 50 \times 1/10 + 500 \times 1/20$ or $0 + 0.5 \times 1/10 + 5 \times 1/20$ $= 30\text{p}$ = £0.30 or $3/10$ Charge “30p” + 20p or 0.3 + 0.2  $= 50\text{p}$ or 0.50 or 0.5	M1 A1 M1  A1 4	or eg 20 goes: $2 \times £0.50 + £5.00$ $= £6.00$ (“£6.00” + $20 \times £0.20$ ) $\div 20$ condone muddled units eg 0.3 + 20  $x = 20, 70, 520$ : M1A1 $20 \times 17/20 + 70 \times 1/10 + 520 \times 1/20$ : M1 $= 50$ : A1  $x, (x - 50), (x - 500)$ : M1A1 $x \times 17/20 + (x - 50) \times 1/10 + (x - 500) \times 1/20 = 20$ : M1 $x = 50$ : A1  Ignore “£” or “p”
<b>Total</b>		<b>7</b>	

<b>5(i)</b>	$\frac{12}{22} \times \frac{11}{21}$ $= \frac{2}{7}$ oe or 0.286 (3 sfs)	M1 A1 2	or ${}^{12}C_2 / {}^{22}C_2$
<b>(ii)</b>	$\frac{7}{15} \times \frac{6}{14} \times \frac{8}{13}$ or $\frac{8}{65}$ oe $\times 3$ oe $= \frac{24}{65}$ or 0.369 (3 sfs)	M1 M1 A1 3	Numerators any order ${}^7C_2 \times {}^8C_1$ :M1 3 x prod any 3 probs (any C or P) ${}^{15}C_3$ :M1 (dep <1)  $1 - (\frac{8}{15} \times \frac{7}{14} \times \frac{6}{13} + 3 \times \frac{8}{15} \times \frac{7}{14} \times \frac{7}{13} + \frac{7}{15} \times \frac{6}{14} \times \frac{5}{13})$ : M2  one prod omitted or wrong: M1
<b>(iii)</b>	$\frac{x}{45} \times \frac{x-1}{44} = \frac{1}{15}$ oe  $x^2 - x - 132 = 0$ or $x(x-1) = 132$  $(x-12)(x+11) = 0$ or $x = \frac{1 \pm \sqrt{(1^2 - 4 \times (-132))}}{2}$ No. of Ys = 12	M1  A1  M1  A1 4	not $\frac{x}{45} \times \frac{x}{44} = \frac{1}{15}$ or $\frac{x}{45} \times \frac{x}{45} = \frac{1}{15}$ or $\frac{x}{45} \times \frac{x-1}{45} = \frac{1}{15}$  oe  ft 3-term QE for M1 condone signs interchanged allow one sign error  Not $x = 12$ or $-11$ ans 12 from less wking, eg $12 \times 11 = 132$ or T & I: full mks  Some incorrect methods:  $\frac{x}{45} \times \frac{x-1}{44} = \frac{1}{15}$ oe M1 $x^2 + x = 132$ A0 $x = 11$ M1A0  $12 \times 11 = 132$ M1A1M1 $x = 12$ and (or "or") 11 A0  NB 12 from eg 12.3 rounded, check method
<b>Total</b>		<b>9</b>	

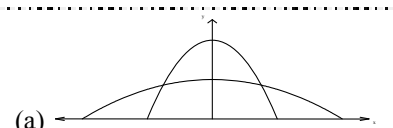
<b>6(i)(a)</b>	256	B1 1	(i)(b) & (ii)(abc): ISW ie if correct seen, ignore extras
<b>(b)</b>	Total unknown or totals poss diff or Y13 may be smaller or similar or size of pie chart may differ	B1 1	pie chart shows only proportions oe or no. of students per degree may differ not "no. of F may be less" not "Y13 may be larger"
<b>(ii)(a)</b>	B&W does not show frequencies oe	B1 1	or B&W shows spread or shows mks or M lger range
<b>(b)</b>	F generally higher or median higher F higher on average or F better mks F IQR is above M IQR  F more compact M wide(r) range or gter IQR or gter variation or gter variance or more spread or less consistent M evenly spread or F skewed	B1          B1 2	1 mk about overall standard, based on median or on F's IQR being "higher"  1 mk about spread (or range or IQR) or about skewness.  must be overall, not indiv mks must be comparison, not just figures  Examples:  not F higher mean     not M have hiest and lowest mks   condone F +ve skew
<b>(c)</b>	<u>Advantage:</u> B&W shows med or Qs or IQR or range or hiest & lowest or key values  <u>Disadvantage:</u> B&W loses info' B&W shows less info' B&W not show freqs B&W not show mode B&W: outlier can give false impression hist shows more info hist shows freqs or fds hist shows modal class (allow mode) hist shows distribution better can calc mean from hist	B1          B1 2	not B&W shows skewness not B&W shows info at a glance not B&W easier to compare data sets not B&W shows mean not B&W shows spread not B&W easier to calculate or easier to read  not B&W does not give indiv (or raw) data not B&W does not show mean  not hist shows freq for each mark not hist shows all the results not hist shows total
<b>(iii)</b>	102 x 51 + 26 x 59 ÷ 128 = 52.6 (3 sfs)	M1 M1dep A1 3	allow adv of hist as disadv of B&W or 5202 + 1534 or 6736
<b>Total</b>		<b>10</b>	

7(i)	Geo stated $0.7^3 \times 0.3$ $\frac{1029}{10000}$ oe or 0.103 (3 sfs)	M1 M1 A1 3	or implied by $0.7^7 \times 0.3$ or $0.3^7 \times 0.7$ Allow $0.7^4 \times 0.3$
(ii)	$0.7^6$ alone = 0.118 (3 sfs)	M1 A1 2	$1 - (0.3 + 0.3 \times 0.7 + \dots + 0.3 \times 0.7^5)$ not $1 - 0.7^6$
(iii)	$0.7^9$ $1 - 0.7^9$ 0.960 (3 sfs)	M1 M1 A1 3	not $0.3 \times 0.7^9$ allow $1 - 0.7^{10}$ or 0.972 for M1 allow 0.96, if no incorrect wking seen  $0.3 + 0.7 \times 0.3 + \dots + 0.7^8 \times 0.3$ : M2 1 term omitted or wrong or "correct" extra: M1
(iv)	Bin stated  ${}^5C_2 \times 0.7^3 \times 0.3^2$ or 0.8369 – 0.5282 = 0.3087 or 0.309 (3 sfs)	M1  M1 A1 3	or implied by table or ${}^nC_r$ or $0.7^3 \times 0.3^2$ or 0.0309
<b>Total</b>		<b>11</b>	
8(i)	$168.6 - \frac{88 \times 16.4}{8}$ $\sqrt{\left(1136 - \frac{88^2}{8}\right)\left(34.52 - \frac{16.4^2}{8}\right)}$ = -0.960 (3 sfs)	M2  A1 3	$\left(= \frac{-11.8}{\sqrt{168 \times 0.9}}\right)$ M1: correct subst in any correct $S$ formula M2: correct substn in any correct $r$ formula  allow -0.96, if no incorrect wking seen
(ii)	must refer to, or imply, external constraint on $x$ e.g $x$ is controlled or values of $x$ fixed or chosen allow $x$ is fixed	B1 1	not $x$ is not random not $x$ affects $y$ not $x$ not affected by $y$ not $x$ goes up same amount each time not charge affects no. of vehicles not $x$ not being measured
(iii)	$168.6 - \frac{88 \times 16.4}{8}$ $1136 - \frac{88^2}{8}$ = -0.0702 (3 sfs) or $^{-59/840}$ or $^{-11.8/168}$  $y - \frac{16.4}{8} = \text{"-0.0702"}(x - \frac{88}{8})$ $y = -0.07x + 2.8$ or better	M1  A1  M1 A1 4	ft their $S_{xy}$ and $S_{xx}$ incl $\frac{168.6}{1136}$ if used in (i)  or -0.07 if no incorrect wking  or $a = \frac{16.4}{8} - (\text{"-0.0702"}) \times \frac{88}{8}$ or $\frac{2371}{840}$ oe eg $y = \frac{-59}{840}x + \frac{2371}{840}$
(iv)(a)	"-0.07" x 20 + "2.8" = 1.4(2) million (2 sfs)	M1 A1 2	no ft
(b)	$r$ close to -1 or corr'n is high         just outside given data, so reliable	B1       B1 2	or good corr'n or pts close to line but not if "close to -1, hence unreliable" if $r$ low in (i), ft: " $r$ low" or "poor corr'n" etc  or outside given data so unreliable  not "reliable as follows trend" not "reliable as follows average" no ft from (iv)(a)
(v)	$y$ on $x$ $x$ is indep	B1 B1 2	or $x$ controlled or $y$ depends on $x$ or $y$ not indep dep on not " $x$ on $y$ "  $r$ close to -1 so makes little difference: B2
<b>Total</b>		<b>14</b>	



# 4733 Probability & Statistics 2

**General:** Conclusions to hypothesis tests must acknowledge uncertainty. Thus “time is unchanged” is A0. Similarly, “Significant evidence that time is unchanged” is also A0.

1	(i)	Biased in favour of those with strong political interest	B2	2	“Biased”, “unrepresentative”, “not indept” or equiv [but <i>not</i> “not random”] stated, with sensible reason. [SR: partial answer, B1]
	(ii)	Obtain list of all pupils Allocate numbers sequentially Choose using random numbers	B1 B1 B1	3	List, can be implied; number serially or randomly, not just “number pupils” Select consistently with method of numbering, not just “select randomly” [SR: systematic: List B1, every $n^{\text{th}}$ B1, random start B1] [SR: names in a hat: B2]
2	(i)	$\Phi\left(\frac{24-30}{12}\right) - \Phi\left(\frac{20-30}{12}\right)$ $= \Phi(-0.5) - \Phi(-0.833)$ $= (1 - 0.6915) - (1 - 0.7976) = \mathbf{0.1061}$	M1 A1 M1 A1	4	Standardise one, allow $\sqrt{12}$ , $12^2$ , $\sqrt{n}$ Both standardisations correct, allow cc here Correct handling of tails [0.3085 – 0.2024] Answer, a.r.t. 0.106, c.a.o.
	(ii)	Not symmetrical (skewed) Therefore inappropriate	M1 A1	2	Any comment implying not symmetric Conclude “not good model” [Partial answer: B1]
3		$H_0 : \mu = 28$ $H_1 : \mu \neq 28$ $\sigma^2 = 37.05 \times 40/39$ [= 38] $z = \frac{26.44 - 28}{\sqrt{38/40}} = -1.601$ Compare $-1.645$ , or $0.0547$ with $0.05$	B2 M1 M1 A1 B1		Both hypotheses correctly stated; one error, allow wrong or no letter, but not $x$ or $t$ or $\bar{x}$ , B1 Multiply $37.05$ or $\sqrt{37.05}$ by $n/(n-1)$ or $\sqrt{[n/(n-1)]}$ Standardise with $\sqrt{n}$ , allow $\sqrt{\quad}$ errors, cc, + Correct $z$ , a.r.t. $-1.60$ , or $p \in [0.0547, 0.0548]$ Explicit comparison of $z$ with $-1.645$ or $p$ with $0.05$
	$\beta$	Critical value $28 - z\sigma/\sqrt{n}$ [= 26.397] $z = 1.645$ Compare $26.44$ with $26.40$	M1 B1 A1 $\sqrt{\quad}$		Allow “ $\pm$ ”, $\sqrt{\quad}$ errors, cc, ignore other tail $z = 1.645$ in CV expression, and compare $26.44$ CV, $\sqrt{\quad}$ on their $z$ , rounding to 3 SF correct
		Do not reject $H_0$ [can be implied] Insufficient evidence that time taken has changed.	M1 A1 $\sqrt{\quad}$	8	Needs $\sqrt{n}$ , correct method & comparison, <i>not</i> $\mu = 26.44$ Conclusion interpreted in context, $\sqrt{\quad}$ on $z$ ,
4	(i)	$\frac{53-50}{\sigma/\sqrt{10}} < 2.326$  $\sigma > \mathbf{4.08}$ <b>AG</b>  [Allow $\geq$ ]	M1 A1 B1 A1	4	Standardise with $10$ or $\sqrt{10}$ and $\Phi^{-1}$ Both sides same sign, $\sqrt{10}$ , don’t worry about $< 2.326$ or $2.33$ seen Convincingly obtain $\sigma > 4.08$ to 3 SF, one other step [SR: Substitution: standardise & substitute $4.08$ M1; $0.0101$ A1; $4.07$ or $4.075$ tried, M1; full justification A1]
	(ii)	$P(\text{Type I}) = 0.01$ used, e.g. Geo( $0.01$ ) $0.99^4 \times 0.01$ $= \mathbf{0.0096}$	M1 M1 A1	3	Not enough merely to state $p = 0.01$ $p^4 \times q$ Answer, a.r.t. $0.0096$
5	(i)	$\int_{-1}^1 \frac{3}{4}(x^2 - x^4)dx = \frac{3}{4} \left[ \frac{x^3}{3} - \frac{x^5}{5} \right]_{-1}^1$ [= 1/5] $1/5 - 0^2$ $= \mathbf{1/5}$	M1 A1 B1 A1	4	Attempt $\int_{-1}^1 x^2 f(x) dx$ Correct indefinite integral Mean 0 clearly indicated Answer $1/5$ or a.r.t. $0.200$ , don’t need $\mu = 0$
	(ii)	 (a) (b) Areas equal, more spread out, so $g_{\max}$ lower (c) $W$ greater as more spread out	B1 M1 A1 B1 dep depB1	5	Correct graph, don’t need $f(x)$ as well. Don’t allow if graph goes further below axis than “pips”. Don’t worry too much about exact shape Mention areas or total probability Convincing argument, not just “flatter” $W$ greater... ...with convincing reason

6	(a)	Po(2.375) $e^{-2.375} \left( \frac{2.375^3}{3!} + \frac{2.375^4}{4!} \right) [= 0.2079 + 0.1233]$ $= \mathbf{0.3310}$	M1 M1 A1 A1	4	Po(19/8) stated or implied One correct Poisson formula, <i>not</i> tables Complete correct expression, including addition Answer, a.r.t. 0.331 [SR: Po(2) or Po(2.4) and tables, M1]
	(b) (i)	$n$ large OR $n > 50$ $p$ small OR $np < 5$	B1 B1	2	Or equivalent [Allow $\leq$ and $\geq$ throughout] Or equivalent, e.g. $np \approx npq$ , or $p < 0.1$ [Treat " $np < 5$ , $npq < 5$ " as single wrong statement]
	(ii)	B(108, $\frac{1}{36}$ ) $\approx$ Po(3) $1 - P(\leq 3) = 1 - 0.6472$ $= \mathbf{0.3528}$	M1 M1 A1 M1 A1	5	Correct binomial distribution stated or implied Po( $np$ ), $\sqrt{\quad}$ on their $n, p$ Po(3) Use Po tables, "1 -", or correct formula, $\pm 1$ term, e.g. 0.1847; a.r.t. 0.353, allow from exact Binomial
7	(i)	Dropped catches must occur independently of one another and at constant average rate	B1 B1	2	"independently", in context, allow "random" "Constant average rate", in context ["Singly" doesn't gain B1]
	(ii)	Use: "Reject $H_0$ when correct" Po(10) $P(\geq 16) = 1 - P(\leq 15) = 1 - 0.9513$  Probability 0.0487	M1 M1 M1  A1		Find $P(\geq r)$ where $r > \lambda$ , e.g. $P(\geq 6)$ from Po(2) Po(10) stated or implied [can be recovered in (iii)] Seek biggest prob $< 0.05$ , e.g. 0.0835 or 0.0166, allow 0.0293 but no other LH tail Answer in range [0.0487, 0.0488], cwd, cwo
		(iii)	$H_0 : \lambda = 10$ or 2 [or $\mu$ ] $H_1 : \lambda > 10$ or 2 [or $\mu$ ] $\alpha : P(\geq 14) = 1 - 0.8645 = 0.1355 > 0.05$  $\beta : \text{Critical region } r \geq 16, p = 0.0487$ Compare $r = 14$	B2  A1 B1  A1√ B1√	
		Do not reject $H_0$ [can be implied] Insufficient evidence of an increase in the number of dropped catches	M1 A1√	10	Method correct, $\sqrt{\quad}$ on $p$ , must be upper tail and " $\geq$ " Conclusion interpreted in context [SR: $P(\leq 14) = 0.9165 < 0.95$ : (B2 M1) A0 B1 M0A0; same for $P(> 14)$ or $P(= 14)$ ] [SR: N(10,10): (ii) 0.05 M0. (iii) (B2) M1 A0 B1 M0A0]
	8	(i)	$H_0 : p = 0.4$ or $\mu = 4.8$ $H_1 : p > 0.4$ or $\mu > 4.8$ B(12, 0.4) $P(\geq 9) = 1 - 0.9847 = 0.0153 < 0.05$ Reject $H_0$ [can be implied] Significant evidence of increase in proportion of audience members who know sponsor's name	B2  M1 A1 B1√ M1 A1√	7
(ii)		N(160, 96)  $\frac{(x-0.5)-160}{\sqrt{96}} = 1.645$  Solve to find $x$ [= 176.6] Minimum value is <b>177</b>	B1 B1 M1 A1 B1 M1 A1	7	Normal, mean 160 Variance (or SD) 96 [96/400: B2M0] Standardise unknown with $np$ and $\sqrt{npq}$ or $npq$ , & equate to $\Phi^{-1}$ ; $\sqrt{96}$ and signs correct, ignore cc RHS = 1.645 Solve [implied by 177 or 176.6 or 176.1] 177 only, from 176.6, CWO [cc error: 6 ex 7]

Marks can be awarded in either part

# 4734 Probability & Statistics 3

1 (i)	$\frac{1}{99}(6115.04 - \frac{761.2^2}{100})$ =3.240	M1 AEF A1 <b>2</b>
<hr/>		
(ii)	$761.2/100 \pm z\sqrt{(3.24/100)}$ $z = 1.96$ (7.26, 7.96)	M1 $z = 1.282, 1.645, \text{ or } 1.96$ B1 A1 <b>3</b> Allow from $\sigma^2 = 3.21$ ; allow 7.97 but not from wrong $\sigma$ . Allow 4 or 5 SF but no more.
<hr/>		
(iii)	None necessary, since sample size large enough for sample mean to have a normal distribution	OR: None necessary, $n$ large enough for Central Limit theorem to apply B1 <b>1</b> <b>[6]</b>
<hr/>		
2	$(\bar{x} - 12.6) / \sqrt{0.1195/10}$  1.383 seen Solve for variable $\bar{x} \geq 12.75$	M1 Any variable, correct mean, /10, ignore $z$ A1 All correct B1 M1 Allow any symbol (<, >, =) A1 <b>5</b> Allow > ; 12.7 or 12.8 No $z$ seen <b>[5]</b>
<hr/>		
3(i)	Choice of newspaper is independent of level of income	B1 <b>1</b> Or equivalent
<hr/>		
----		
(ii)	Use $df=4$ EITHER: CV 13.28, from $df=4$ or sig. level Largest significance level is 1% OR: Use $P(\chi^2 > 12.32)$ Largest significance level is 1.52%	B1 May be implied by 13.28 seen or 0.0152 M1 From tables B1 Accept 0.01 Use of calculator B2 <b>3</b> Accept 0.0152 <b>[4]</b> SR: from $df=6$ : CV 12.59 used ; SL=5% : <b>B0M1B1</b>
<hr/>		
4(i)	$\int_0^1 \frac{4}{3} x^3 dx + \int_1^2 \frac{4}{3x^3} dx$ Limits seen anywhere	M1 For both integrals OR $1 - \int_2^\infty \frac{4}{3x^3} dx$ A1 For both OR $1 - \left[ -\frac{2}{3x^2} \right]_2^\infty$ A1 <b>3</b>
<hr/>		
--		
(ii)	EITHER: $\int_0^1 \frac{4}{3} x^3 dx = \frac{1}{3}$ $< 1/2$ Median must exceed 1 OR: $m = \sqrt{(4/3)}$ $> 1$ AG	M1 A1 A1 M1 Attempt to find median A1 M0 for $1.5^{1/4}$ A1 <b>3</b> Accept 1.15..

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(iii)	$\int_0^1 \frac{4}{3} x^4 dx + \int_1^\infty \frac{4}{3x^2} dx$ $[4x^5/15] + [-4/(3x)]$ 1.6	M1 B1 A1	Correct form for at least one integral Both integrals correct without limits 3 AEF
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(iv)	$E(X^2) = \dots + \int_1^\infty \frac{4}{3x} dx$ Second integral = $\left[ \frac{4}{3} \ln x \right]_1^\infty$ This is not finite, (so variance not finite)	M1 A1 A1	For second integral  3 AEF [12]
-----			
5 (i)	Justify a relevant Poisson approximation $E(A) = 75 \times 0.022 (=1.65)$ , $E(B) = 90 \times 0.025 (=2.25)$ Sum of two independent Poisson variables $X$ has a Poisson distribution Mean $m = 3.9$	M1 B1B1 A1 B1	Using $n > 50$ or $n$ large; $np < 5$ or $p$ small ( $< 0.1$ ) or $np \approx npq$  5 Accept Po(3.9)
-----			
(ii)	$1 - P(\leq 5)$  0.1994	M1 A1	Or From Po( $m$ ) Accept $\leq 4$ ; OR Exact $1 -$ sum of at least 5 correct terms 2 From calculator or tables, art 0.20 [7]
-----			
6 (i)	Use $p_s \pm zs$ $z = 2.326$ $s = \sqrt{(0.12 \times 0.88/50)}$ $(0.013, 0.227)$ Allow limits if penalised in Q1	M1 B1 A1 A1	Or /49 4 Or (0.012, 0.228) from 49
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---			
(ii)	$z(0.12 \times 0.88/n)^{1/2}$ $< 0.05$ Solve to obtain $n > 228.5$ $n \approx 229$ or 230	M1 A1 M1 A1 A1	Any $z$ Allow = Must contain $\sqrt{n}$ Accept = 5 Must be integer [9]
-----			
7 (i)	Each population of test scores should have normal distributions with equal variances	B1 B1	OR: Variances equal and normal distns Context 2 B1 B1
-----			
(ii)	EITHER: Cannot test for normality from data OR: Sample variances are close enough to accept population variances equal	B1	Not variances are not equal 1
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<p>(iii) <math>H_0: \mu_B = \mu_G, H_1: \mu_B &gt; \mu_G</math>  <math>s^2 = (23 \times 86.79 + 17 \times 93.01) / 40</math>  <math>= 89.4335</math>  <math>t = (1238.4 / 18 - 1526.8 / 24) / [s^2(18^{-1} + 24^{-1})]^{1/2}</math>  <math>= 1.758</math>            Use CV of 1.684  <math>1.758 &gt; 1.684</math>            Reject <math>H_0</math> and accept there is sufficient evidence at the 5% significance level that teenage boys worry more, on average than teenage girls.</p>	<p>B1 For both. No other variables. Allow words            M1 Finding pooled estimate of variance            A1 May be implied by later value of <math>t</math>            M1 With pooled estimate of variance            A1 All correct            A1 art 1.76, or -            B1 Consistent            M1 Compare correctly with their CV (<math>t</math> value)              Not assertive              A1√ 9 Ft on their 1.758            SR: Using <math>s^2 = 93.01/18 + 86.79/24</math> :            B1M0A0M1A0A1 (for 1.749) B1M1 (from 1.645 or 1.684) A1            Max 6/9</p>
<b>12]</b>	
<p>8 (i) <math>\Sigma xf / 80 = 1.9</math> AG  <math>\Sigma x^2 f / 80 - 1.9^2</math>            1.365 or 1.382</p>	<p>B1 With evidence            M1 Or <math>\times 80/79</math>            A1 3</p>
<p>(ii) Poisson distribution requires equal mean and variance            EITHER:            No, mean and variance differ significantly            OR:            Yes, indicated by sample statistics taking into account sampling error</p>	<p>B1 May be indicated                B1 2</p>
<p>(iii) <math>e^{-1.9} 1.9^3 / 3!</math>  <math>\times 80</math></p>	<p>B1 Or from tables            B1 2</p>
<p>(iv) Considering sample as random selection of all similar matches  <math>H_0</math>: Poisson suitable model            Combine last two cells  <math>0.97^2 / 11.97 + 7.73^2 / 22.73 + 11.40^2 / 21.60</math>  <math>+ 2.32^2 / 13.68 + 5.02^2 / 10.02</math>  <math>= 11.63</math>            CV 7.815  <math>11.63 &gt; 7.815</math>              There is sufficient evidence that a Poisson distribution is not a suitable model confirming (or not) the answer to part (ii)</p>	<p>B1              B1            M1 Any two correct            A1 All correct            A1 art 11.6            B1 *dep OR <math>p = 0.00875</math>            M1dep* OR <math>0.00875 &lt; 0.05</math>                A1√ 8 Ft (ii)            SR: If last cells not combined: <math>\chi^2 = 12.3</math>            M1A1A1 CV=9.448 or <math>p = 0.0152</math>, B1*dep the M1dep*</p>
<p>(v) E-values or probabilities would change            df would increase by 1</p>	<p>B1 Or other valid observation            B1 2 Or CV would change            [17]</p>

# 4735 Statistics 4

1 (i)	Use $P(A) + P(B) - P(A \cap B) \leq 1$ , $P(A \cap B) = 0$	B1	1	AEF
-----				
(ii)	Use $P(A B) = P(A \cap B) / P(B)$ Use $P(A \cap B) = 0$ with argument with $x \neq 0$	M1 A1	2	AEF e.g. Independent if $(A \cap B) = P(A)P(B) = x^2$ , $P(A \cap B) = 0$ , $x \neq 0$ , so $A$ and $B$ are not indep.
-----				
(iii)	Use $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$ Use $P(A \cap B) = 0$ ; $P(A \cap B \cap C) = 0$ $P(B \cap C) = 2x^2$ ; $P(C \cap A) = 2x^2$ Substitute and obtain required result	M1 A1 A1 AG	4 (7)	Or equivalent. Allow one sign error For both For both
-----				
2 (i)	Wilcoxon test requires a symmetric distribution not supported by the diagram	B1	1	Or equivalent
-----				
(ii)	$H_0: m = 1.80$ , $H_1: m > 1.80$ Use sign test Number exceeding 1.8 = 20 Use $B(30, 0.5)$ , $P(\geq 20)$ Or $P(\leq 10)$ 0.0494 Compare with 0.05 correctly 2.008 Conclude there is significant evidence that the median time exceeds 1.80 sec	M1 A1 M1 A1 M1 A1√	7 (8)	B1 Needs "population median" if words OR: 1.645 if $N(15, 7.5)$ , $z = 1.643$ , 1.816, used; OR CR ( $X \geq 20$ ) fit $p$ or $z$
-----				
3 (i)	Marginal distribution of $X$ $x$ 0 1 2 3 $p$ 0.27 0.23 0.32 0.18 $1 \times 0.23 + 2 \times 0.32 + 3 \times 0.18$ $= 1.41$	B1 M1 A1	3	
-----				
(ii)	$P(Y > X) = 0.08 + 0.05 + 0.03 + 0.08 + 0.06 + 0.07$ $= 0.37$	M1 A1	2	
-----				
(iii)	Use $P(Y > X \cap X > 0) / P(X > 0)$ $P(X > 0) = 0.73$ $P(Y > X \cap X > 0) = 0.08 + 0.06 + 0.07$ $21/73$	M1 A1 A1 A1	4	From marginal distribution AEF
-----				
(iv)	The director cannot conclude independence from cov. So director's conclusion incorrect. OR: Eg $P(X=0 \cap Y=0) = 0.11$ , $P(X=0)P(Y=0) = 0.27 \times 0.29 \neq P(X=0 \cap Y=0)$	M1 A1 M1 A1	2 (11)	Idea that independence implies $\text{cov} = 0$ but not the reverse

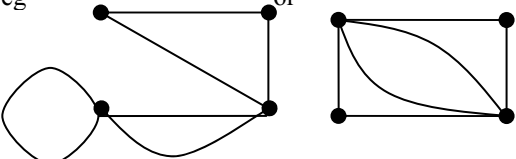
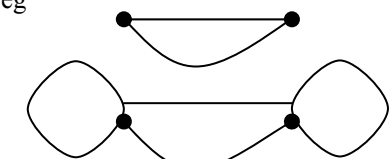
4 (i)	Variances seem not to be equal	B1	1	
-----				
(ii)	$H_0: m_M = m_A, H_1: m_M \neq m_A$ “average”		B1	Both hypotheses, AEF. Not
	$R_m = 40, m(m+n+1) - R_m = 72$	M1		Both found
	$W = 40$	A1		A0 if no or wrong 72
	CR: $W \leq 38$	B1		
	40 not in CR, so do not reject $H_0$	M1		Or equivalent
	Insufficient evidence that median times differ	A1	6 (7)	In context. B1 if no M1 but conclusion correct Allow average here
-----				
5 (i)	$a+b = 3/4$ $M'(0) = 3^3/8$ $1/2 + 3a + 4b = 3^3/8$ Solve simultaneously $a = 1/8$ AG $b = 5/8$	B1 M1 A1 M1 A1 A1		From $M(0)=1$ AEF Elimination or substitution
-----				
(ii)	$M''(t) = e^{2t} + 9/8 e^{3t} + 10e^{4t}$ $M''(0) - (M'(0))^2$ $97/8 - (3^3/8)^2$ ; $47/64$	B1 M1 A1A1	4	
-----				
(iii)	$x = 2, 3, 4$	B1	1 (11)	
-----				
6 (i)	$P(Y > y) = 1 - F(y)$ $= a^3/y^3$ $P(S > s) = P(\text{all 3 values } > s) = (a/s)^9$ AG $f(s) = d/ds(1 - (a/s)^9)$ $= \begin{cases} 9 \frac{a^9}{s^{10}} & s \geq a, \\ 0 & s < a \end{cases}$	M1 A1 A1 M1 A1		Allow any variables
-----				
(ii)	$\int_a^\infty \frac{a^9}{s^9} ds$ $= 9a/8$ $S$ not unbiased since this not equal to $a$ $T_1 = 8S/9$	M1 A1 M1 B1√	4	Ft $E(S)$
-----				
(iii)	$\text{Var}(T_1) = a^2/63, \text{Var } T_2 = a^2/9$ $\text{Var}(T_1) < \text{Var}(T_2), T_1$ is more efficient	M1 A1 for both A1√	3	Correct method Comparison, completion.. √ one variance correct with same dimensions
-----				
(iv)	$t_1 = 4.0, t_2 = 5.4$ From data $a \leq 4.5$ and $t_2 > 4.5$	B1 B1B1	3 (15)	Both AEF

7 (i)	$G(1) = 1$ $a = 2$	M1 A1	2
<hr/>			
(ii)	$(1+2t)/(4-t) = c(1+2t)(1-\frac{1}{4}t)^{-1}$ $= \frac{1}{4}(1+2t)(1 + \frac{1}{4}t + (\frac{1}{4}t)^2 + \dots)$ Coefficient of $t^3 = \frac{1}{4}[(\frac{1}{4})^3 + 2(\frac{1}{4})^2]$ $= \frac{9}{256}$	M1 A1 M1√	$c = \frac{1}{4}$ or 4  With 2 terms from previous line A1 4
<hr/>			
(iii)	$H(t) = \left(\frac{1+2t}{4-t}\right)^3$  $H'(t) = 3\left(\frac{1+2t}{4-t}\right)^2 \left[\frac{2(4-t)+1+2t}{(4-t)^2}\right]$  $E(Y) = H'(1)$ $= 3$	B1  M1A1  M1 A1	5
<hr/>			
(iv)	$H(1) = p_0 + p_1 + p_2 + p_3 + p_4 + \dots = 1$ $H(-1) = p_0 - p_1 + p_2 - p_3 + p_4 - \dots = -\frac{1}{125}$ Add: $2(p_0 + p_2 + p_4 + \dots) = 1 - \frac{1}{125}$ $\frac{1}{2}(1 - \frac{1}{125})$ AG	M1 A1	With sufficient detail 2 (13)



# 4736 Decision Mathematics 1

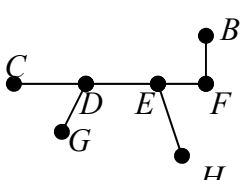
1	(i)	Biggest/largest/last number (only) (Not showing effect on a specific list)	B1	Accept bubbling to left unless inconsistent with part (ii): Smallest/first number	[1]
	(ii)	2 1 3 4 5 horizontally or vertically (may see individual comparisons/swaps) [For reference: original list was 3 2 1 5 4] 4 comparisons and 3 swaps (both correct)	M1 A1	Or bubbling to left: 1 3 2 4 5 Watch out for shuttle sort used  If not stated, assume that comparisons come first	[2]
	(iii)	1 2 3 4 5  One (more pass after this)	M1 A1	FT from their first pass with their bubbling if possible Watch out for 'One swap (in 2 <sup>nd</sup> pass)'	[2]
	(iv)	$(3000 \div 500)^2 \times 0.2$  = 7.2 seconds	M1 A1	$6^2 \times 0.2$ or $8 \times 10^{-7} \times 9 \times 10^6$ or any equivalent calculation cao UNITS	[2]
<b>Total = 7</b>					

2	(i)	eg   - Graph is not simple - Two of the vertices are joined by two arcs (if appropriate) - It has a 'loop' (if appropriate) - For a simple graph each vertex must have order 3 or less	M1 A1 B1	A graph with four vertices of orders 2, 2, 4, 4 (ignore any vertex labels) A connected graph  Recognition that their graph is not simple (although it is connected). Need not use the word 'simple'.	[3]
	(ii)	eg   Graph is not connected	M1 A1 B1	Any graph with four vertices of orders 2, 2, 4, 4 (that is topologically different from that in part (i)) A graph that is not connected  Recognition in words that their graph is not connected	[3]
<b>Total = 6</b>					

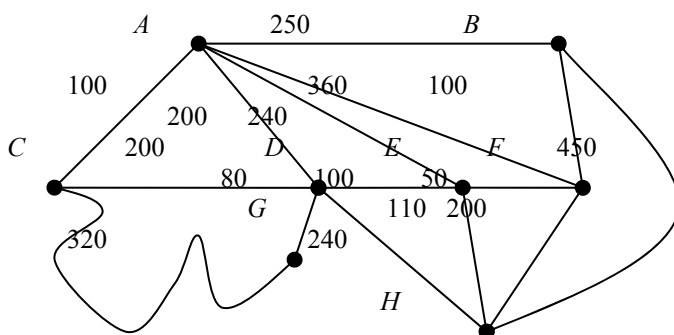
3	(i)	$y \leq x + 2$ $x + 2y \geq 6$ ( $y \geq -\frac{1}{2}x + 3$ ) $2x + y \leq 12$ ( $y \leq -2x + 12$ )	M1 M1 M1 A1	Line $y = x + 2$ in any form Line $x + 2y = 6$ in any form Line $2x + y = 12$ in any form All inequalities correct	[4]
	(ii)	$x + 2y = 6$ and $y = x + 2 \Rightarrow (\frac{2}{3}, 2\frac{2}{3})$ $y + 2x = 12$ and $y = x + 2 \Rightarrow (3\frac{1}{3}, 5\frac{1}{3})$ $y + 2x = 12$ and $x + 2y = 6 \Rightarrow (6, 0)$	M1 A1 A1 B1	Follow through if possible Calculating from their lines or implied from either A mark $(\frac{2}{3}, \frac{8}{3})$ (art (0.7, 2.7)) $(\frac{10}{3}, \frac{16}{3})$ (art (3.3, 5.3)) $(6, 0)$ cao	[4]
	(iii)	$(\frac{2}{3}, 2\frac{2}{3}) \Rightarrow 11\frac{1}{3}$ $(3\frac{1}{3}, 5\frac{1}{3}) \Rightarrow 32\frac{2}{3}$ $(6, 0) \Rightarrow 30$ At optimum, $x = 3\frac{1}{3}$ and $y = 5\frac{1}{3}$ Maximum value = $32\frac{2}{3}$	M1 A1 A1	Follow through if possible Testing vertices or using a line of constant profit (may be implied) Accept $(3\frac{1}{3}, 5\frac{1}{3})$ identified (ft) $32\frac{2}{3}$ (air 32.6 to 32.7) (ft)	[3]
	(iv)	$5 \times 3\frac{1}{3} + k \times 5\frac{1}{3} \geq 5 \times 6 + k \times 0$ $\Rightarrow k \geq 2.5$	M1 M1 A1	$5 \times 3\frac{1}{3} + k \times 5\frac{1}{3}$ (ft) or implied $5 \times 6 + k \times 0$ or 30 or implied Greater than or equal to 2.5 (cao)	[3]
<b>Total =</b>					<b>14</b>

<p>4 (i)</p>	<div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <table border="1" style="margin-bottom: 5px;"> <tr><td>1</td><td>0</td></tr> <tr><td colspan="2"> </td></tr> </table> <p><i>A</i></p> </div> <div style="text-align: center;"> <table border="1" style="margin-bottom: 5px;"> <tr><td>4</td><td>5</td></tr> <tr><td>6</td><td>5</td></tr> </table> <p><i>B</i></p> </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 20px;"> <div style="text-align: center;"> <table border="1" style="margin-bottom: 5px;"> <tr><td>5</td><td>6</td></tr> <tr><td>6</td><td> </td></tr> </table> <p><i>C</i></p> </div> <div style="text-align: center;"> <table border="1" style="margin-bottom: 5px;"> <tr><td>(9)</td><td>(16)</td></tr> <tr><td>16</td><td> </td></tr> </table> <p><i>F</i></p> </div> <div style="text-align: center;"> <table border="1" style="margin-bottom: 5px;"> <tr><td>7</td><td>12</td></tr> <tr><td>12</td><td> </td></tr> </table> <p><i>H</i></p> </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 20px;"> <div style="text-align: center;"> <table border="1" style="margin-bottom: 5px;"> <tr><td>3</td><td>3</td></tr> <tr><td>4</td><td>3</td></tr> </table> <p><i>D</i></p> </div> <div style="text-align: center;"> <table border="1" style="margin-bottom: 5px;"> <tr><td>2</td><td>2</td></tr> <tr><td>2</td><td> </td></tr> </table> <p><i>E</i></p> </div> <div style="text-align: center;"> <table border="1" style="margin-bottom: 5px;"> <tr><td>6</td><td>10</td></tr> <tr><td>10</td><td> </td></tr> </table> <p><i>G</i></p> </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 20px;"> <div style="text-align: center;"> <table border="1" style="margin-bottom: 5px;"> <tr><td>(10)</td><td>(16)</td></tr> <tr><td>16</td><td> </td></tr> </table> <p><i>J</i></p> </div> <div style="text-align: center;"> <table border="1" style="margin-bottom: 5px;"> <tr><td>8</td><td>14</td></tr> <tr><td>14</td><td> </td></tr> </table> <p><i>K</i></p> </div> </div> <p style="margin-top: 20px;">Route = <math>A - E - B - G - H - K</math> Length = 14 metres</p>	1	0			4	5	6	5	5	6	6		(9)	(16)	16		7	12	12		3	3	4	3	2	2	2		6	10	10		(10)	(16)	16		8	14	14		<p>M1 Both 6 and 5 shown at <i>B</i></p> <p>M1 All temporary labels correct including <i>F</i> and <i>J</i></p> <p>A1 No extra temporary labels</p> <p>B1 All permanent labels correct (may omit <i>F</i> and/or <i>J</i>) cao</p> <p>B1 Order of labelling correct (may omit <i>F</i> and/or <i>J</i>, may reverse <i>F</i> and <i>J</i>) cao</p> <p>B1 <math>A - E - B - G - H - K</math> cao</p> <p>B1 14 cao</p>	<p>[7]</p>
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(10)	(16)																																										
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14																																											
	<p>(ii) Without using <i>CJ</i>: Route = <math>A - E - B - G - F - J</math> Length = 21 metres</p>	<p>B1 Follow through their (i)</p> <p>B1 <math>A - E - B - G - F - J</math></p> <p>B1 21</p>	<p>[2]</p>																																								
	<p>(iii) More than 2 metres</p> <p>(Answer of 'more than 7 metres' or '7 metres' <math>\Rightarrow</math> M1, A0)</p>	<p>M1 2 (cao)</p> <p>A1 More than, or equivalent</p> <p>(Answer of 3 or <math>\geq 3 \Rightarrow</math> SC1)</p>	<p>[2]</p>																																								
<p><b>Total = 11</b></p>																																											

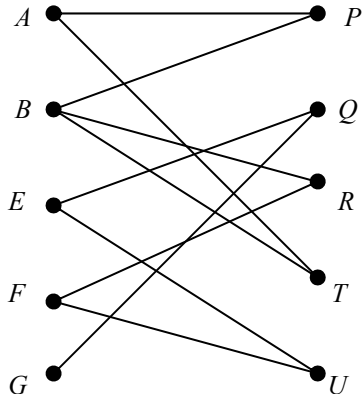
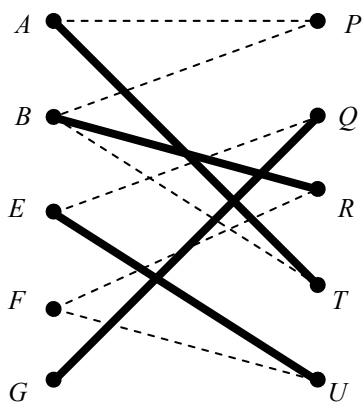
5	(i)		<i>E</i>	<i>W</i>	B1 B1 B1 M1 A1	$AW = 3 - x$ $BW = 3 - y$ $CE = 4 - x - y$ , in any form  An appropriate calculation for their table  Leading to given result	[5]			
		<i>A</i>	$x$	$3 - x$						
		<i>B</i>	$y$	$3 - y$						
		<i>C</i>	$4 - x - y$	$x + y - 1$						
		$\begin{aligned} \text{Total cost} &= \pounds(250x + 250(3-x) \\ &\quad + 200y + 140(3-y) \\ &\quad + 300(4-x-y) + 280(x+y-1)) \\ &= \pounds(2090 - 20x + 40y) \quad (\text{AG}) \end{aligned}$								
	(ii)	$\begin{aligned} 2090 - 20x + 40y &\leq 2150 \\ \Rightarrow -20x + 40y &\leq 60 \\ \Rightarrow -x + 2y &\leq 3 \quad (\text{AG}) \end{aligned}$			B1	Showing where the given inequality comes from	[1]			
	(iii)	$\begin{aligned} 50(3-x) + 40(3-y) + 60(x+y-1) \\ = 210 + 10x + 20y \\ \text{So need to maximise } x + 2y \quad (\text{AG}) \end{aligned}$			M1 A1	Follow through their table Correct expression $210 + 10x + 20y$	[2]			
	(iv)	<i>P</i>	<i>x</i>	<i>y</i>	<i>s</i>	<i>t</i>	-	B1 B1	Rows and columns may be in any order $-1 -2$ in objective row Constraint rows correct	[2]
		1	-1	-2	0	0	0			
		0	-1	2	1	0	3			
		0	1	1	0	1	3			
	(v)	Pivot on the 2 in the <i>y</i> column					B1	$\begin{matrix} 1 & -2 & 0 & 1 & 0 & 3 \\ 0 & -0.5 & 1 & 0.5 & 0 & 1.5 \\ 0 & 1.5 & 0 & -0.5 & 1 & 1.5 \end{matrix}$	[6]	
		1	-2	0	1	0				3
		0	-0.5	1	0.5	0				1.5
		0	1.5	0	-0.5	1	1.5	M1 A1	Pivot row correct Other rows correct	
		Pivot on 1.5 in the <i>x</i> column					M1 A1 B1	$\begin{matrix} 1 & 0 & 0 & \frac{1}{3} & 1\frac{1}{3} & 5 \\ 0 & 0 & 1 & \frac{1}{3} & \frac{1}{3} & 2 \\ 0 & 1 & 0 & -\frac{1}{3} & \frac{2}{3} & 1 \end{matrix}$	[6]	
		1	0	0	$\frac{1}{3}$	$1\frac{1}{3}$				5
		0	0	1	$\frac{1}{3}$	$\frac{1}{3}$				2
		0	1	0	$-\frac{1}{3}$	$\frac{2}{3}$	1			
		$x = 1, y = 2$								
<b>Total = 16</b>										

6	(a)(i)	Route Inspection (problem)	B1	Or Chinese postman (problem)	[1]
	(ii)	<p>Odd nodes are <math>A, B, C</math> and <math>D</math></p> <p><math>AB = 250</math>    <math>AC = 100</math>    <math>AD = 200</math>  <math>CD = \underline{200}</math>    <math>BD = \underline{250}</math>    <math>BC = \underline{350}</math>                    450            350            550</p> <p>Repeat <math>AC</math> and <math>BFED = 350</math>                  Length of shortest route = 3350 metres</p>	B1 M1 A1 B1	<p>Identifying odd nodes (may be implied from working)</p> <p>Pairing odd nodes (all three pairings considered)</p> <p>M mark may not be implied</p> <p>350 as minimum</p> <p>3350 m or 3.35 km    UNITS</p>	[4]
	(iii)	<p><math>C</math> is an odd node, so we can end at another odd node.</p> <p><math>AB = 250</math>    <math>AD = 200</math>    <math>BD = 250</math>                  Repeat <math>AD = 200</math>                  Length of route = 3200 metres                  Route ends at <math>B</math></p>	M1 A1 B1	<p>Working need not be seen</p> <p>May be implied from answer</p> <p>3200</p> <p><math>B</math></p>	[3]
	(b)(i)	<p><math>D - G - C - A - E - F - B - H - D</math></p> <p>1580 metres</p> <p><math>A - C - D - G</math> then method stalls</p>	M1 A1 B1	<p>Correct cycle</p> <p>If drawn then arcs must be directed</p> <p>1580</p> <p>Identifying the stall</p>	[3]
	(ii)	<p><math>BF = 100</math>  <math>FE = 50</math>  <math>ED = 100</math>  <math>DG = 80</math>  <math>EH = 110</math>  <math>DC = 200</math></p>  <p>Order of adding nodes: <math>B F E D G H C</math>                  Total weight of tree = 640 metres</p>	M1 A1 B1 A1 B1	<p>Use of Prim's algorithm to build tree (e.g. an attempt at list of arcs or order of adding vertices). NOT Kruskal</p> <p>Correct arcs chosen (listed or seen on tree)</p> <p>A correct tree with vertices labelled</p> <p>Order stated or clearly implied</p> <p>640</p>	[5]
	(iii)	<p>Lower bound = <math>640 + 100 + 200 = 940</math>                  940 metres <math>\leq</math> shortest tour <math>\leq</math> 1580 metres</p>	M1 A1	<p><math>300 +</math> weight of their tree</p> <p>their <math>940 \leq</math> length <math>\leq</math> their 1580</p> <p>(condone use of <math>&lt;</math> here)</p>	[2]
<b>Total = 18</b>					

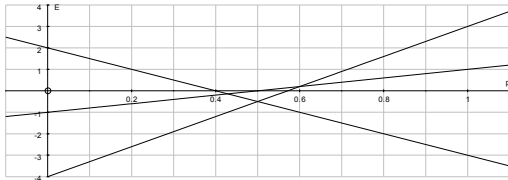
For reference:



# 4737 Decision Mathematics 2

1(a)	(i)		B1	A correct bipartite graph	[1]																																		
	(ii)		B1	A second bipartite graph showing the incomplete matching correctly	[1]																																		
	(iii)	<p><math>F - R - B - P</math></p> <p><math>A = T \quad B = P \quad E = U \quad F = R \quad G = Q</math></p>	B1	This path in any reasonable form	[2]																																		
	(iv)	<p><math>A = P \quad B = T \quad E = U \quad F = R \quad G = Q</math></p>	B1	This complete matching	[1]																																		
(b)	(i)	<p>Hungarian algorithm finds the minimum cost matching, subtract from 10 to convert a maximising problem into a minimising problem.</p> <p>Column <math>X</math> is a dummy column (dummy task) to make the table square</p>	B1	An appropriate reference to maximising/minimising	[2]																																		
	(ii)	<table border="1" data-bbox="311 1489 845 1691"> <thead> <tr> <th></th> <th><math>C</math></th> <th><math>D</math></th> <th><math>L</math></th> <th><math>S</math></th> <th><math>X</math></th> </tr> </thead> <tbody> <tr> <th><math>H</math></th> <td>1</td> <td>2</td> <td>4</td> <td>4</td> <td>10</td> </tr> <tr> <th><math>I</math></th> <td>2</td> <td>4</td> <td>7</td> <td>6</td> <td>10</td> </tr> <tr> <th><math>J</math></th> <td>4</td> <td>6</td> <td>5</td> <td>9</td> <td>10</td> </tr> <tr> <th><math>K</math></th> <td>3</td> <td>8</td> <td>7</td> <td>7</td> <td>10</td> </tr> <tr> <th><math>N</math></th> <td>3</td> <td>7</td> <td>7</td> <td>5</td> <td>10</td> </tr> </tbody> </table>		$C$		$D$	$L$	$S$	$X$	$H$	1	2	4	4	10	$I$	2	4	7	6	10	$J$	4	6	5	9	10	$K$	3	8	7	7	10	$N$	3	7	7	5	10
	$C$	$D$	$L$	$S$	$X$																																		
$H$	1	2	4	4	10																																		
$I$	2	4	7	6	10																																		
$J$	4	6	5	9	10																																		
$K$	3	8	7	7	10																																		
$N$	3	7	7	5	10																																		
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	<p>Reduce columns</p> <table border="1" style="margin-left: 20px;"> <tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>2</td><td>3</td><td>2</td><td>0</td></tr> <tr><td>3</td><td>4</td><td>1</td><td>5</td><td>0</td></tr> <tr><td>2</td><td>6</td><td>3</td><td>3</td><td>0</td></tr> <tr><td>2</td><td>5</td><td>3</td><td>1</td><td>0</td></tr> </table> <p>Rows are already reduced</p> <p>Augment by 1</p> <table border="1" style="margin-left: 20px;"> <tr><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>2</td><td>1</td><td>0</td></tr> <tr><td>2</td><td>3</td><td>0</td><td>4</td><td>0</td></tr> <tr><td>1</td><td>5</td><td>2</td><td>2</td><td>0</td></tr> <tr><td>1</td><td>4</td><td>2</td><td>0</td><td>0</td></tr> </table> <p><math>H = D</math> Harry is the director  <math>I = C</math> Iannos operates the camera  <math>J = L</math> Jack is in charge of lighting  <math>N = S</math> Nadia is in charge of sound          (and Kerry is not used)</p> <p>Total score =  <math>(10-2) + (10-2) + (10-5) + (10-5) + (10-10)</math>  <math>= 26</math></p>	0	0	0	0	0	1	2	3	2	0	3	4	1	5	0	2	6	3	3	0	2	5	3	1	0	0	0	0	0	1	0	1	2	1	0	2	3	0	4	0	1	5	2	2	0	1	4	2	0	0	<p>M1 Either reducing columns or reducing rows of <math>5 \times 5</math> matrix</p> <p>A1 This reduced matrix Correct answer only</p> <p>M1 A reasonable attempt to augment</p> <p>A1 This final matrix Correct answer only</p> <p>B1 This matching, indicated in any way</p> <p>M1 A reasonable attempt, 14 or 24 <math>\Rightarrow</math> M1, A0 26</p>	<p>[7]</p>																																							
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<p>(iii)</p>	<table border="1" style="margin-left: 20px;"> <tr><td></td><td><i>C</i></td><td><i>D</i></td><td><i>L</i></td><td><i>S</i></td></tr> <tr><td><i>I</i></td><td>2</td><td>4</td><td>7</td><td>6</td></tr> <tr><td><i>J</i></td><td>4</td><td>6</td><td>5</td><td>9</td></tr> <tr><td><i>K</i></td><td>3</td><td>8</td><td>7</td><td>7</td></tr> <tr><td><i>N</i></td><td>3</td><td>7</td><td>7</td><td>5</td></tr> </table> <p>Reduce columns</p> <table border="1" style="margin-left: 20px;"> <tr><td>0</td><td>0</td><td>2</td><td>1</td></tr> <tr><td>2</td><td>2</td><td>0</td><td>4</td></tr> <tr><td>1</td><td>4</td><td>2</td><td>2</td></tr> <tr><td>1</td><td>3</td><td>2</td><td>0</td></tr> </table> <p>Then reduce rows</p> <table border="1" style="margin-left: 20px;"> <tr><td>0</td><td>0</td><td>2</td><td>1</td></tr> <tr><td>2</td><td>2</td><td>0</td><td>4</td></tr> <tr><td>0</td><td>3</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>3</td><td>2</td><td>0</td></tr> </table> <p><math>I = D</math>    <math>J = L</math>    <math>K = C</math>    <math>N = S</math></p>		<i>C</i>	<i>D</i>	<i>L</i>	<i>S</i>	<i>I</i>	2	4	7	6	<i>J</i>	4	6	5	9	<i>K</i>	3	8	7	7	<i>N</i>	3	7	7	5	0	0	2	1	2	2	0	4	1	4	2	2	1	3	2	0	0	0	2	1	2	2	0	4	0	3	1	1	1	3	2	0	<p>B1 This <math>4 \times 4</math> matrix (need not have row and column labels)</p> <p>Or reduce rows</p> <table border="1" style="margin-left: 20px;"> <tr><td>0</td><td>2</td><td>5</td><td>4</td></tr> <tr><td>0</td><td>2</td><td>1</td><td>5</td></tr> <tr><td>0</td><td>5</td><td>4</td><td>4</td></tr> <tr><td>0</td><td>4</td><td>4</td><td>2</td></tr> </table> <p>M1</p> <p>Then reduce columns</p> <table border="1" style="margin-left: 20px;"> <tr><td>0</td><td>0</td><td>4</td><td>2</td></tr> <tr><td>0</td><td>0</td><td>0</td><td>3</td></tr> <tr><td>0</td><td>3</td><td>3</td><td>2</td></tr> <tr><td>0</td><td>2</td><td>3</td><td>0</td></tr> </table> <p>A1</p> <p>B1 This matching indicated in any way Correct answer only</p>	0	2	5	4	0	2	1	5	0	5	4	4	0	4	4	2	0	0	4	2	0	0	0	3	0	3	3	2	0	2	3	0	<p>[4]</p>
	<i>C</i>	<i>D</i>	<i>L</i>	<i>S</i>																																																																																								
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<p><b>Total = 18</b></p>																																																																																												

2	(i)	-2	B1	Accept 'loses 2' or equivalent	[1]																																			
	(ii)	Column $W$ is dominated by column $Y$ . If Rowena plays $P$ , Collette loses 2 with $W$ but 1 with $Y$ . If Rowena plays $Q$ , Collette loses 1 with $W$ but gains 1 with $Y$ .	B1 B1	Stating $Y$ (but not $W$ dominates $Y$ ) Correct comparisons explained, $2 > 1$ and $1 > -1$ , or equivalent	[2]																																			
	(iii)	<table style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td></td> <td colspan="4" style="text-align: center;">Collette</td> <td></td> </tr> <tr> <td></td> <td></td> <td style="text-align: center;">[<math>W</math>]</td> <td style="text-align: center;"><math>X</math></td> <td style="text-align: center;"><math>Y</math></td> <td style="text-align: center;"><math>Z</math></td> <td></td> </tr> <tr> <td style="text-align: right;">Rowena</td> <td style="text-align: center;"><math>P</math></td> <td style="text-align: center;">[2]</td> <td style="text-align: center;">-3</td> <td style="text-align: center;">1</td> <td style="text-align: center;">3</td> <td style="text-align: right;">Row min -3</td> </tr> <tr> <td></td> <td style="text-align: center;"><math>Q</math></td> <td style="text-align: center;">[1]</td> <td style="text-align: center;">2</td> <td style="text-align: center;">-1</td> <td style="text-align: center;">-4</td> <td style="text-align: right;">-4</td> </tr> <tr> <td style="text-align: right;">Col max</td> <td></td> <td style="text-align: center;">[2]</td> <td style="text-align: center;">2</td> <td style="text-align: center;">1</td> <td style="text-align: center;">3</td> <td></td> </tr> </table> <p>Play-safe for Rowena is <math>P</math> Play-safe for Collette is <math>Y</math></p>			Collette							[ $W$ ]	$X$	$Y$	$Z$		Rowena	$P$	[2]	-3	1	3	Row min -3		$Q$	[1]	2	-1	-4	-4	Col max		[2]	2	1	3		M1 A1 A1	Determining row minima and column maxima, or equivalent. Must be correct, including $W$ if shown. May not be implied from answers. $P$ stated $Y$ stated	[3]
		Collette																																						
		[ $W$ ]	$X$	$Y$	$Z$																																			
Rowena	$P$	[2]	-3	1	3	Row min -3																																		
	$Q$	[1]	2	-1	-4	-4																																		
Col max		[2]	2	1	3																																			
	(iv)	$-3p + 2(1-p) = 2-5p$ $Y$ gives $2p-1$ $Z$ gives $7p-4$	B1 B1	$2-5p$ in simplified form Both $2p-1$ and $7p-4$ in any form	[2]																																			
	(v)	 <p><math>7p - 4 = 2 - 5p \Rightarrow p = 0.5</math> <math>E = -0.5</math></p>	B1 M1 A1 B1	Graph must be on graph paper Their lines drawn correctly on a reasonable scale Solving the correct pair of equations (only) or using graph correctly 0.5, correct answer only -0.5, correct answer only	[4]																																			
	(vi)	Add 4 throughout matrix to make all values non-negative On this augmented matrix, if Collette plays $Y$ Rowena expects $4p_1 + 3p_2 + 6p_3$ , and if Collette plays $Z$ Rowena expects $7p_1 + 0p_2 + 2p_3$  We are solving a <u>maximin</u> problem. $m$ is less than or equal to each of these values since we need find the maximum value of the worst possible augmented expected pay-off for each value of $p$	B1 B1 B1	'Add 4', or new matrix written out or equivalent Relating to columns $Y$ and $Z$ respectively. Note: $4p_1 + 3p_2 + 6p_3$ and $7p_1 + 2p_3$ are given in question Or shown on a diagram. For <u>each value of <math>p</math></u> we look at the minimum output.	[3]																																			
	(vii)	We use an inequality instead of an equality because this is needed to enable the Simplex algorithm to pivot on a row that will increase the value of $M$	B1	So that we can use the Simplex algorithm.	[1]																																			
	(viii)	$p_3 = \frac{3}{7}$ $E = \frac{6}{7}$	B1 B1	$\frac{3}{7}$ $\frac{6}{7}$	[2]																																			
<b>Total = 18</b>																																								



ANSWERED ON INSERT

3	(i)	$\{S, A, B, D, G\}, \{C, E, F, T\}$ (given) $AC = 4, BC = 2, BE = 1, DE = 2, GE = 5, GT = 6$  $4+2+1+2+5+6$ $= 20$ litres per minute	M1	Identifying the correct arcs, on a diagram or list or by using 4, 2, 1, 2, 5, 6	
			A1	20 from a correct calculation	[2]
	(ii)	At most 2 litres per minute can enter $G$ so the arc $GE$ can carry at most 2 litres per minute	B1	Maximum into $G = 2$	[1]
	(iii)	At most 8 litres per minute can flow into $E$  Flow shown on diagram on insert Flow in = flow out for each vertex except $S, T$  A feasible flow of 8 litres per minute through $E$	B1	8	
			M1	A flow of the rate they have claimed through $E$ (irrespective of whether it is feasible) (directions may not be changed, assume a blank means 0)	
			A1	No pipe capacities exceeded and flow through $E = 8$	[3]
	(iv)	Arrows labelled on diagram $SA = 0 \quad AC = 0 \quad CF = 0 \quad FT = 1$ $AS = 4 \quad CA = 4 \quad FC = 4 \quad TF = 4$  $AB = 3 \quad BC = 2 \quad CE = 3 \quad EF = 4$ $BA = 0 \quad CB = 0 \quad EC = 0 \quad FE = 0$  $SB = 4 \quad BE = 0 \quad ET = 5$ $BS = 1 \quad EB = 1 \quad TE = 1$  $BD = 3 \quad DE = 2 \quad EG = 0$ $DB = 0 \quad ED = 0 \quad GE = 5$  $SD = 0 \quad DG = 0 \quad GT = 4$ $DS = 2 \quad GD = 2 \quad TG = 2$	M1	Assume blanks mean 0 Arrows on arcs on one of the routes $SACFT, SBET, SDGT$ labelled correctly, or all labels on the route reversed	
			M1	Arrows on all three routes labelled correctly or all reversed	
		A1	All arrows labelled correctly, not reversed	[3]	
(v)	Amount that flows along $SBDET = 2$ litres per min  $SB = 4 \ 2 \quad BD = 3 \ 1 \quad DE = 2 \ 0 \quad ET = 5 \ 3$ $BS = 1 \ 3 \quad DB = 0 \ 2 \quad ED = 0 \ 2 \quad TE = 1 \ 3$	B1	2		
		M1	For arrows on route $SBDET$ : Labels updated consistently		
		A1	These all labelled correctly (and not reversed)	[3]	
(vi)	Route used = $SBCET$  $SB = 4 \ 2 \ 0 \quad BC = 2 \ 0 \quad CE = 3 \ 1 \quad ET = 5 \ 3$ $1$ $BS = 1 \ 3 \ 5 \quad CB = 0 \ 2 \quad EC = 0 \ 2 \quad TE = 1 \ 3$ $5$	B1	$SBCET$ listed		
		M1	For arrows on route $SBCET$ : Labels updated consistently		
		A1	These all labelled correctly (and not reversed)	[3]	
(vii)		B1	Follow through their (v) and (vi) if possible Assume blanks mean 0	[1]	
(viii)	Eg cut through arcs $SA, SB, SD$ Or arcs $AC, BC, BE, DE, DG$	M1	A suitable cut chosen, indicated in any way		
		A1	Indicated by listing arcs cut	[2]	
<b>Total = 18</b>					

PART (a) ANSWERED ON INSERT

4	(a)	<table border="1"> <thead> <tr> <th>Stage</th> <th>State</th> <th>Action</th> <th>Working</th> <th>Suboptimal maximum</th> </tr> </thead> <tbody> <tr> <td rowspan="3">2</td> <td>0</td> <td>0</td> <td>5</td> <td>5</td> </tr> <tr> <td>1</td> <td>0</td> <td>4</td> <td>4</td> </tr> <tr> <td>2</td> <td>0</td> <td>4</td> <td>4</td> </tr> <tr> <td rowspan="6">1</td> <td rowspan="2">0</td> <td>0</td> <td><math>3 + 5 = 8</math></td> <td>8</td> </tr> <tr> <td>1</td> <td><math>4 + 4 = 8</math></td> <td>8</td> </tr> <tr> <td rowspan="2">1</td> <td>1</td> <td><math>2 + 4 = 6</math></td> <td rowspan="2">8</td> </tr> <tr> <td>2</td> <td><math>4 + 4 = 8</math></td> </tr> <tr> <td rowspan="2">2</td> <td>1</td> <td><math>6 + 4 = 10</math></td> <td>10</td> </tr> <tr> <td>2</td> <td><math>5 + 4 = 9</math></td> <td></td> </tr> <tr> <td rowspan="3">0</td> <td rowspan="3">0</td> <td>0</td> <td><math>4 + 8 = 12</math></td> <td rowspan="3">13</td> </tr> <tr> <td>1</td> <td><math>5 + 8 = 13</math></td> </tr> <tr> <td>2</td> <td><math>2 + 10 = 12</math></td> </tr> </tbody> </table> <p>Length of longest path = 13 Route = (0;0) – (1;1) – (2;2) – (3;0)</p>	Stage	State	Action	Working	Suboptimal maximum	2	0	0	5	5	1	0	4	4	2	0	4	4	1	0	0	$3 + 5 = 8$	8	1	$4 + 4 = 8$	8	1	1	$2 + 4 = 6$	8	2	$4 + 4 = 8$	2	1	$6 + 4 = 10$	10	2	$5 + 4 = 9$		0	0	0	$4 + 8 = 12$	13	1	$5 + 8 = 13$	2	$2 + 10 = 12$	<p>B1 5, 4, 4 identified as suboptimal maxima for stage 2</p> <p>M1 Transferring suboptimal maxima from stage 2 to stage 1 correctly</p> <p>A1 Correct additions or totals seen for all rows in stage 1</p> <p>B1 8, 8, 10 identified as suboptimal maxima for stage 1 (cao)</p> <p>M1 Transferring suboptimal maxima from stage 1 to stage 0 correctly</p> <p>A1 Correct additions or totals seen for all rows in stage 0</p> <p>B1 13</p> <p>B1 Correct route or in reverse (including (0; 0) and (3; 0))</p>	[8]
		Stage	State	Action	Working	Suboptimal maximum																																														
2	0	0	5	5																																																
	1	0	4	4																																																
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(b)(i)		<p>M1 Condone directions missing Must be activity on arc</p> <p>A1 A reasonable attempt, arcs should be labelled</p> <p>A1 Any correct form Condone extra dummies provided precedences are not violated, accept networks with multiple end vertices Arc weights may be shown but are not necessary</p>	[2]																																																	
(ii)	<p>Minimum project completion time = 13 days Critical activities B, G, L</p>	<p>M1 Follow through their network if possible Values at vertices may be recorded using any consistent notation</p> <p>A1 Forward pass with no more than one independent error Forward pass correct</p> <p>M1 Backward pass with no more than one independent error (follow through their 13)</p> <p>A1 Backward pass correct</p> <p>B1 13 stated, cao</p> <p>B1 B, G, L correct answer only</p>	[6]																																																	
(iii)		<p>B1 Not follow through</p> <p>B1 A directed dummy from end of G to start of K</p> <p>B1 A directed dummy from end of G to start of L</p> <p>Condone extra dummies provided precedences are not violated Watch out for K following I</p>	[2]																																																	
<b>Total = 18</b>																																																				

# Grade Thresholds

Advanced GCE Mathematics (3890-2, 7890-2)  
June 2008 Examination Series

## Unit Threshold Marks

7892		Maximum Mark	A	B	C	D	E	U
4721	Raw	72	63	55	47	39	32	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	56	49	42	35	29	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	55	47	40	33	26	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	56	49	43	37	31	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	49	43	37	31	25	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	54	47	41	35	29	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	61	53	45	37	29	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	56	47	38	29	20	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	56	47	38	29	21	0
	UMS	100	80	70	60	50	40	0
4731	Raw	72	59	50	42	34	26	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	60	52	45	38	31	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	55	48	41	34	28	0
	UMS	100	80	70	60	50	40	0
4735	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	53	46	39	32	26	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	61	54	47	40	34	0
	UMS	100	80	70	60	50	40	0

## Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
<b>3890</b>	300	240	210	180	150	120	0
<b>3891</b>	300	240	210	180	150	120	0
<b>3892</b>	300	240	210	180	150	120	0
<b>7890</b>	600	480	420	360	300	240	0
<b>7891</b>	600	480	420	360	300	240	0
<b>7892</b>	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
<b>3890</b>	33.3	50.4	65.4	77.0	86.6	100	14679
<b>3891</b>	100	100	100	100	100	100	1
<b>3892</b>	57.2	76.7	88.2	94.1	97.6	100	1647
<b>7890</b>	45.4	67.3	82.4	92.1	97.8	100	10512
<b>7891</b>	33.3	66.7	100	100	100	100	6
<b>7892</b>	56.5	77.9	90.0	95.4	98.2	100	1660

For a description of how UMS marks are calculated see:

[http://www.ocr.org.uk/learners/ums\\_results.html](http://www.ocr.org.uk/learners/ums_results.html)

Statistics are correct at the time of publication.

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